Review Problems for Quiz #7

1. Use the technique of Separation of Variables to solve each of the following differential equations. Your answers may contain unspecified constants.

(a) $y' = \dfrac{x^2}{y}$ **(b)** $y' + y^2 \cdot \sin(x) = 0$

(c) $y' = \dfrac{\cos^2(x)}{\cos^2(2y)}$ **(d)** $y' = \dfrac{x - e^{-x}}{y + e^{y}}$

2. Use integrating factors to solve each of the following initial value problems. Your answers should not contain any unspecified constants.

(a) $y' - y = 2t \cdot e^{2t}$ $y(0) = 1.$

(b) $t \cdot y' + 2y = t^2 - t + 1$ $y(1) = \frac{1}{2}.$

(c) $y' - 2y = e^{2t}$ $y(0) = 2.$

(d) $t^3 \cdot y' + 4t^2 \cdot y = e^{-t}$ $y(-1) = 0.$

3. Newton's Law of Cooling states that the rate at which an object cools (or heats up) is proportional to the difference in temperature between the object and its surroundings. Suppose a cup of coffee has an initial temperature of 200°F when freshly poured. The cup is in a room that has a temperature of 70°F. One minute after it was poured, the coffee reaches a temperature of 190°F. How long does it take for the cup of coffee to reach 150°F?

4. A dead body is discovered at midnight. When it is discovered, an investigator determines that the temperature of the body is 85°F. The body was in a room that had a temperature of 68°F and it remains in that room as the investigation into the cause of death takes place. Two hours later, the investigator takes the temperature of the body again and finds that it is now (at 2am) 74°F. When, approximately, did the person die?

5. A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ grams per liter of salt enters the tank at a rate of 2 liters per minute and the well-stirred mixture leaves the tank at the same rate. Find a formula for the mass of salt in the tank (in grams) t minutes after the salty solution started flowing. Your answer should contain the constant γ but no other constants.

HINT: If you are having a hard time seeing how to remove the constant of integration, think about how much salt is in the tank initially.

6. Solve each of the following initial value problems. Your final answers should not contain any unspecified constants.

(a) $y'' + y' - 2y = 0 \qquad y(0) = 1 \qquad y'(0) = 1.$

(b) $6y'' - 5y' + y = 0 \qquad y(0) = 4 \qquad y'(0) = 0.$

(c) $y'' + 5y' + 3y = 0 \qquad y(0) = 1 \qquad y'(0) = 0.$

(d) $y'' + 8y' - 9y = 0 \qquad y(1) = 1 \qquad y'(1) = 0.$

7. Solve each of the following initial value problems. Your final answers should not contain any unspecified constants.

(a) $y'' + y' - 2y = 2t \qquad y(0) = 0 \qquad y'(0) = 1.$

(b) $y'' - 2y' - 3y = 3t \cdot e^{2t} \qquad y(0) = 1 \qquad y'(0) = 0.$

Answers

1.(a) $y = \pm\sqrt{\dfrac{C + 2x^3}{3}}.$

1.(b) $y = \pm\sqrt{\dfrac{C + 2 \cdot \ln\left(\left|1 + x^3\right|\right)}{3}}.$

1.(c) $y = \dfrac{1}{C - \cos(x)}.$

1.(d) The following equation implicitly defines y as a function of x. (It is possible to solve for y using the quadratic formula – see if you can figure out how.)

$$3y + y^2 - x^3 + x = C.$$

2.(a) $y = 3e^t + 2(t-1)e^{2t}.$

2.(b) $y = \dfrac{3t^4 - 4t^3 + 6t^2 + 1}{12t^2}.$

2.(c) $y = (t+2)\cdot e^{2t}.$

2.(d) $y = \dfrac{-(t+1)\cdot e^{-t}}{t^4}.$

3. $t = \dfrac{\ln\left(\frac{13}{8}\right)}{\ln\left(\frac{13}{12}\right)} \approx 6.07$ minutes.

4. The person probably died at about 10:50pm.

5. Use $M(t)$ for the mass of salt after t minutes. Then: $M(t) = 120\cdot\gamma - 120\cdot\gamma\cdot e^{-\frac{t}{60}}.$

6.(a) $y = e^t.$

6.(b) $y = 12e^{\frac{t}{3}} - 8e^{\frac{t}{2}}.$

6.(c) $y = \dfrac{13+5\sqrt{13}}{26}\cdot e^{\frac{(-5+\sqrt{13})t}{2}} + \dfrac{13-5\sqrt{13}}{26}\cdot e^{\frac{(-5-\sqrt{13})t}{2}}.$

6.(d) $y = \frac{e^9}{10}\cdot e^{-9t} + \frac{9}{10e}\cdot e^t.$

7.(a) $y = e^t - \frac{1}{2}e^{-2t} - t - \frac{1}{2}.$

7.(b) $y = e^{3t} + \frac{2}{3}e^{-t} - \frac{2}{3}e^{2t} - t\cdot e^{2t}.$