Additional Review Problems[1] for Quiz #5

1. Find the mass of a rod of length 10 cm with density $\delta(x) = e^{-x}$ g/cm at a distance of x cm from the left end of the rod.

2. A rod has a length of 2 meters. At a distance of x meters from its left end, the density of the rod is given by:

$$\delta(x) = 2 + 6x \text{ g/m.}$$

(a) Write a left hand Riemann sum approximating the total mass of the rod using N rectangles.

(b) Find the exam mass of the rod.

3. The density of cars (in cars per mile) down a 20-mile stretch of the Pennsylvania turnpike is approximated by:

$$\delta(x) = 300\left(2 + \sin\left(4\sqrt{x + 0.15}\right)\right).$$

(a) Write a left hand Riemann sum that approximates the total number of cars on this 20-mile stretch, using N "rectangles."

(b) Find the total number of cars in the 20-mile stretch of road. (It is okay to use your calculator to evaluate this integral.)

4. A lobster tank in a restaurant is 4 feet long by 3 feet wide by 2 feet deep. Find the force on the bottom and on each of the four sides in units of pounds. The density of water is 62.4 pounds per cubic foot.

5. On August 12, 2000, the Russian submarine Kursk sank to the bottom of the ocean, approximately 120 meters below the surface of the water. You can assume that the density of the seawater was 1027 kg/m^3. Find the following:

(a) The water pressure at the bottom of the ocean where the Kursk sank.

(b) The force on a square metal plate (2 meters by 2 meters) resting horizontally on the bottom of the sea.

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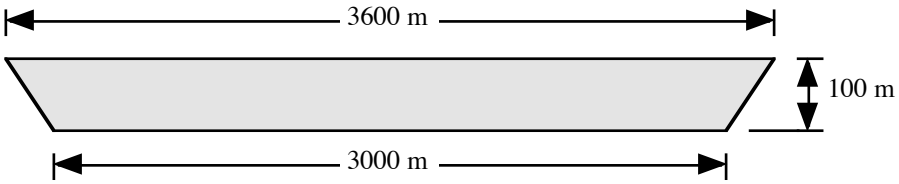
[1] The problems given here are adapted from *Calculus* by Gleason, Hughes-Hallet et al.

(c) The force on a square metal plate (2 meters by 2 meters) that is suspended vertically with its bottom on the sea floor.

6. Set up and evaluate a definite integral giving the total force on the dam shown in the figure below (which is about the same size as the Aswan Dam in Egypt). You can assume that the water contained by the dam is fresh water with a density of 1000 kg/m^3.



Answers

1. Mass = $1 - e^{-10}$ g.

2.(a) Let $\Delta x = \frac{2-0}{N}$. Then the total mass will be approximated by: $\sum_{k=0}^{N-1} \delta(0 + k \cdot \Delta x) \cdot \Delta x$, where $\delta(x) = 2 + 6x$.

2.(b) Mass = 16 g.

3.(a) Let $\Delta x = \frac{20-0}{N}$. Then the total number of cars will be approximated by: $\sum_{k=0}^{N-1} \delta(0 + k \cdot \Delta x) \cdot \Delta x$, where $\delta(x) = 300\left(2 + \sin\left(4\sqrt{x + 0.15}\right)\right)$.

3.(b) Approximately 11513 cars.

4. Bottom: 1497.6 pounds.
Shorter sides: 374.4 pounds.
Longer sides: 499.2 pounds.

5.(a) Pressure = 1207752 kg/m^2.

5.(b) Force = 4831008 N.

5.(c) Force = 4790749.2 N.

6. Total force = 1.6×10^{11} N.