SOLUTIONS



## Quiz #3

Questions 1-4 are all long-response questions. In each case, be careful to indicate your final answer **and show how you obtained it**. Answers with no supporting work will get no credit.

You may assume the following integral formula on this quiz.

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

**You should not use your calculator for any of the problems on this quiz.**

**1.** **(2 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \frac{1}{x^2+x}\,dx.$$

$$\int \frac{1}{x^2+x}\,dx = \int \frac{1}{x(x+1)}\,dx.$$

Use partial fractions to express $\frac{1}{x(x+1)}$.

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$= \frac{A(x+1) + Bx}{x(x+1)}$$

Equating coefficients of powers of $x$:

$$A + B = 0$$

$$A = 1$$

so $A = 1$ and $B = -1$.

$$\int \frac{1}{x^2+x}\,dx = \int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx$$

$$= \ln(|x|) - \ln(|x+1|) + C$$

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**2.** **(3 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \frac{1}{x^3+4x^2+4x}\,dx.$$

$$\int \frac{1}{x^3+4x^2+4x}\,dx = \int \frac{1}{x(x^2+4x+4)}\,dx$$

$$= \int \frac{1}{x(x+2)^2}\,dx$$

Use partial fractions to express $\frac{1}{x(x+2)^2}$.

$$\frac{1}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$= \frac{A(x+2)^2 + Bx(x+2) + Cx}{x(x+2)^2}$$

Equating coefficients of powers of x in numerators:

$\underline{x^2:}$ $\quad A + B = 0$

$\underline{x^1:}$ $\quad 4A + 2B + C = 0$

$\underline{x^0:}$ $\quad 4A = 1$

So, $A = 1/4$, $B = -1/4$, $C = -1/2$.

$$\int \frac{1}{x^3+4x^2+4x}\,dx = \int \frac{1/4}{x}\,dx - \int \frac{1/4}{x+2}\,dx + \int \frac{-1/2}{(x+2)^2}\,dx$$

$$= \frac{1}{4}\ln(|x|) - \frac{1}{4}\ln(|x+2|)$$

$$+ \frac{1}{2}(x+2)^{-1} + C.$$

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**3.** **(3 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \frac{x^4 - 3x^3 + 3x^2 - 3x + 3}{x^2 + 1}\,dx.$$

Since the degree of the polynomial in the numerator is greater than the degree of the polynomial in the denominator, simplify using polynomial long division.

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 + 1 & x^4 - 3x^3 + 3x^2 - 3x + 3 \\
 & \underline{x^4 \qquad\qquad + x^2} \\
 & \quad -3x^3 + 2x^2 \\
 & \quad \underline{-3x^3 \qquad\quad - 3x} \\
 & \qquad\qquad 2x^2 \\
 & \qquad\qquad \underline{2x^2 \qquad\quad + 2} \\
 & \qquad\qquad\qquad\qquad 1
\end{array}
$$

So,

$$\frac{x^4 - 3x^3 + 3x^2 - 3x + 3}{x^2 + 1} = x^2 - 3x + 2 + \frac{1}{x^2 + 1}$$

So,

$$\int \frac{x^4 - 3x^3 + 3x^2 - 3x + 3}{x^2 + 1}\,dx = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + \arctan(x) + C$$

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**4.** **(2 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \frac{1}{x^2-8x+17}\,dx.$$

The denominator is a quadratic that does not factor. We will complete the square on the denominator and attempt to apply

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C.$$

$x^2 - 8x + 17$ Coefficient of $x$ is $-8$
Half of coefficient is $-4$
Square of this is $16$.

$= x^2 - 8x + 16 - 16 + 17$ "Giveth and taketh away"

$= (x-4)^2 - 16 + 17$ Factor first three terms

$= (x-4)^2 + 1$ Combine constants.

So,
$$\int \frac{1}{x^2-8x+17}\,dx = \int \frac{1}{1+(x-4)^2}\,dx$$

let $u = x-4$
$du = dx$

$$= \int \frac{1}{1+u^2}\,du$$

$$= \arctan(u) + C$$

$$= \arctan(x-4) + C$$