SOLUTIONS

Math 122 **Fall 2008**

Quiz #2

Questions 1-4 are all long-response questions. In each case, be careful to indicate your final answer **and show how you obtained it**. Answers with no supporting work will get no credit.

You may assume the following trigonometric identities on this quiz.

(a) $\sin^2(x)+\cos^2(x)=1$ (b) $\tan^2(x)+1=\sec^2(x)$

(c) $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$ (d) $\cos^2(x)=\frac{1}{2}(1+\cos(2x))$

You may assume the following integral formulas on this quiz.

(a) $\int \tan(x)dx=\ln(|\sec(x)|)+C$ (b) $\int \sec(x)dx=\ln(|\sec(x)+\tan(x)|)+C$

You should not use your calculator for any of the problems on this quiz.

1. **(2 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \frac{\sin(x)+2}{\cos(x)}dx.$$

$$\int \frac{\sin(x)+2}{\cos(x)}dx = \int \frac{\sin(x)}{\cos(x)}dx + 2\int \frac{1}{\cos(x)}dx$$

$$= \int \tan(x)\,dx + 2\int \sec(x)\,dx$$

$$= \ln(|\sec(x)|) + 2\cdot\ln(|\sec(x)+\tan(x)|)+C$$

SOLUTIONS

2. **(3 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \tan^5(x)dx.$$

$$\int \tan^5(x)\,dx = \int \tan(x)\cdot \tan^4(x)\,dx$$

$$= \int \tan(x)\cdot \left(\sec^2(x) - 1\right)^2 \cdot dx$$

$$= \int \tan(x)\cdot \left(\sec^4(x) - 2\sec^2(x) + 1\right) dx$$

$$= \int \tan(x)\cdot \sec(x)\cdot \sec^3(x)\,dx$$

$$- 2\int \tan(x)\cdot \sec(x)\cdot \sec(x)\,dx$$

$$+ \int \tan(x)\,dx$$

$$= \frac{1}{4}\sec^4(x) - \sec^2(x)$$

$$+ \ln\left(|\sec(x)|\right) + C$$

SOLUTIONS

3. **(3 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int x\sqrt{1-x^4}\,dx.$$

Start with the substitution: $w = x^2.$

$$\frac{dw}{dx} = 2x$$

$$dx = dw/2x.$$

$$\int x\sqrt{1-x^4}\,dx = \int x\sqrt{1-w^2}\,dw/2x$$

$$= \frac{1}{2}\int\sqrt{1-w^2}\,dw$$

Now make the trig substitution: $w = \sin(\theta)$

$$dw = \cos(\theta)\,d\theta.$$

$$\int\sqrt{1-w^2}\,dw = \int\sqrt{1-\sin^2(\theta)}\cdot\cos(\theta)\cdot d\theta$$

$$= \int\cos^2(\theta)\,d\theta$$

$$= \int\frac{1}{2}(1+\cos(2\theta))\,d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C$$

Now, $\theta = \sin^{-1}(w)$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$= 2w\cdot\sqrt{1-w^2}$$

So: $\int\sqrt{1-w^2}\,dw = \frac{1}{2}\sin^{-1}(w) + \frac{1}{4}\cdot 2\cdot w\sqrt{1-w^2} + C$

and:

$$\int x\sqrt{1-x^4}\,dx = \frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + C.$$

SOLUTIONS.

4. **(2 points)** Evaluate the following indefinite integral to find the most general antiderivative.

$$\int \frac{1}{x\sqrt{x^2+9}}\,dx.$$

Let $x = 3\cdot\tan(\theta)$. Then:

$$\frac{dx}{d\theta} = 3\cdot\sec^2(\theta)$$

so:

$$dx = 3\cdot\sec^2(\theta)\,d\theta.$$

$$\int \frac{1}{x\sqrt{x^2+9}}\,dx = \int \frac{1}{3\cdot\tan(\theta)\sqrt{9\tan^2(\theta)+9}}\cdot 3\sec^2(\theta)\,d\theta$$

$$= \int \frac{3\sec^2(\theta)}{9\cdot\tan(\theta)\cdot\sec(\theta)}\,d\theta$$

$$= \frac{1}{3}\int \frac{\sec(\theta)}{\tan(\theta)}\,d\theta$$

Now,

$$\frac{\sec(\theta)}{\tan(\theta)} = \frac{1}{\cos(\theta)}\cdot\frac{\cos(\theta)}{\sin(\theta)} = \frac{1}{\sin(\theta)}$$

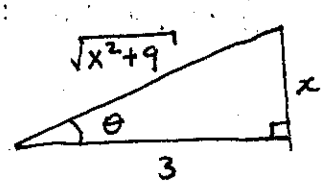
$$= \csc(\theta)$$

$$\int \frac{1}{x\sqrt{x^2+9}}\,dx = \frac{1}{3}\int \csc(\theta)\,d\theta$$

$$= \frac{1}{3}\ln\left|\csc(\theta) - \cot(\theta)\right| + C$$

$$= \frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}}{x} - \frac{3}{x}\right| + C$$

Note: $\csc(\theta) = \dfrac{\sqrt{x^2+9}}{x}$ comes from:



$\tan(\theta) = \dfrac{x}{3}$.

$$\csc(\theta) = \frac{\text{hyp.}}{\text{opp.}}$$

$$= \frac{\sqrt{x^2+9}}{x}.$$