Unit Test 1 Review Problems – Set B

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. In this problem you are required to calculate formulas for each of the antiderivatives (or indefinite integrals) listed below. Each of these problems can be solved using the technique of u-substitution, although you are welcome to use whatever methods you choose. Your answers should each include an unspecified constant along the lines of "+*C*."

(a) $\int s\cdot\sqrt{s+1}\cdot ds$ **(b)** $\int \frac{x^2+x}{\sqrt{x+1}}\cdot dx$

(c) $\int x^2\cdot\sqrt{x-2}\cdot dx$ **(d)** $\int \frac{p}{\sqrt{p+1}}\cdot dp$

2. Evaluate the integrals (a)-(d) given below. All that you can assume about the function $f(x)$ is that it is a continuous function for $0 \le x \le 6$ with that it has the following properties:

$$f(0)=2,\qquad f(2)=3,\qquad f(4)=-1,\qquad f(6)=5$$

$$f'(0)=1,\qquad f'(2)=4$$

$$\int_0^2 f(x)dx=3,\qquad \int_2^4 f(x)dx=1,\qquad \int_4^6 f(x)dx=6.$$

(a) $\int_0^2 xf'(x)dx$ **(b)** $\int_2^4 f'(x)\big(2+3f(x)\big)dx$

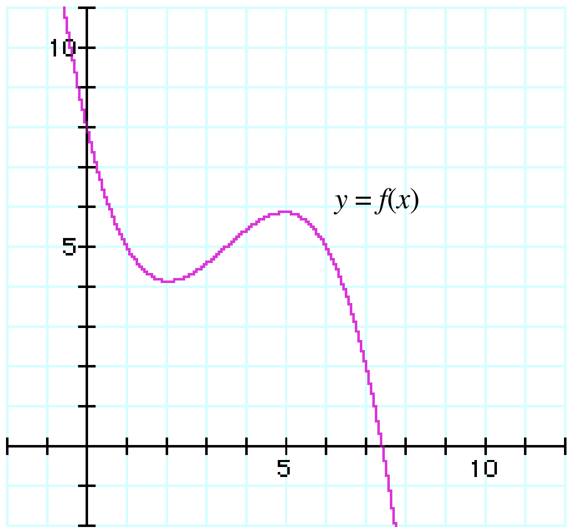
(c) $\int_0^2 f(3x)dx$ **(d)** $\int_0^2 x\cdot f(x^2)dx$

3. In this problem your goal is to evaluate each of the integrals given below. Use techniques such as partial fractions, polynomial long division and completing the square to accomplish this. Do not use calculator integration to find the antiderivatives.

(a) $\int \frac{3x^2 - 8x + 1}{x^3 - 4x^2 + x + 6}\,dx$ **(b)** $\int \frac{10x + 2}{x^3 - 5x^2 + x - 5}\,dx$

(c) $\int \frac{x^4 + 3x^3 + 2x^2 + 1}{x^2 + 3x + 2}\,dx$ **(d)** $\int \frac{e^x}{(e^x - 1)(e^x + 2)}\,dx$

4. In this problem, the function $f(x)$ will always refer to the function defined by the graph shown below.



(a) Use the graph of $y = f(x)$ provided above to sketch the area corresponds to the symbolic statement $\sum_{k=0}^{3} f\left(1 + k \cdot \frac{3}{2}\right) \cdot \frac{3}{2}$.

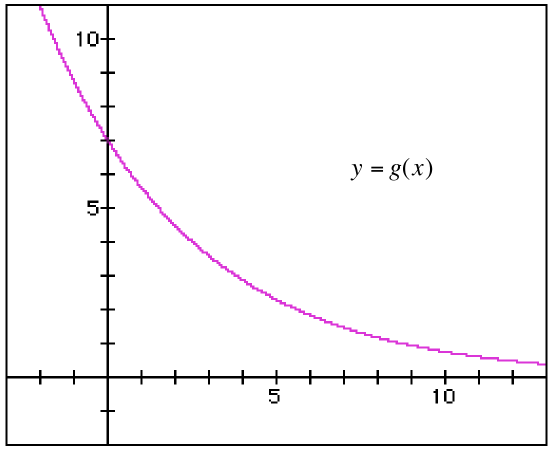
(b) Consider the following sum expressed in sigma notation: $\sum_{k=0}^{3} f\left(1 + k \cdot \frac{3}{2}\right) \cdot \frac{3}{2}$. What Riemann sum does this correspond to?

(c) The table (below) gives some of the values of the function $f(x)$. Use the values of $f(x)$ provided in the table to find the numerical value of the sum:

$$\sum_{k=0}^{3} f\left(1 + k \cdot \tfrac{3}{2}\right) \cdot \tfrac{3}{2}.$$

x	$f(x)$	x	$f(x)$
0	8	4	5.25
0.5	6	4.5	5.8
1	5	5	5.9
1.5	4.3	5.5	5.6
2	4.1	6	5
2.5	4.3	6.5	3.5
3	4.5	7	2
3.5	5	7.5	−0.2

(d) The function $g(x)$ is defined by the graph shown below.



Arrange the following five quantities from smallest to largest.

(I) $\sum_{k=0}^{3} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}$ (II) $\sum_{k=1}^{4} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}$ (III) $\int_{1}^{7} g(x)dx$

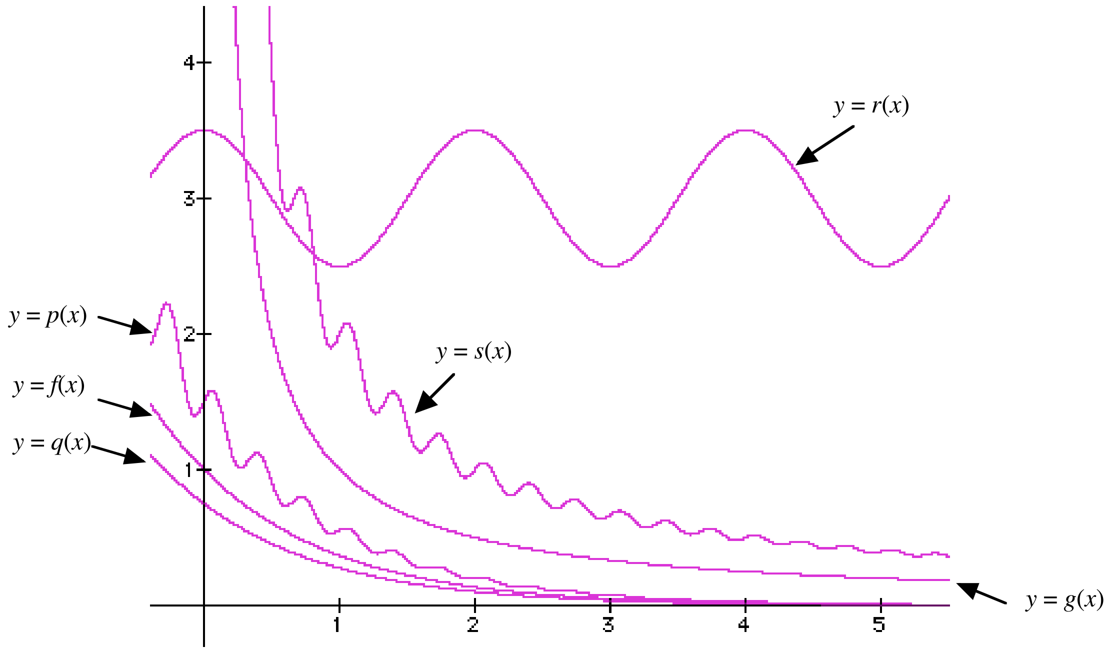
(IV) $\dfrac{\sum_{k=0}^{3} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}+\sum_{k=1}^{4} g(1+k\cdot\frac{3}{2})\cdot\frac{3}{2}}{2}$ (V) $\sum_{k=0}^{3} g(1+\frac{3}{4}+k\cdot\frac{3}{2})\cdot\frac{3}{2}$

5. Your goal in this problem is to evaluate each of the integrals shown below. The methods of trigonometric substitution are likely to be helpful here. When you write out the integral and make the substitution, don't forget to write the *dx* in your integral. If you don't you may miss some of the factors that need to be present for you to solve each problem correctly.

(a) $\int \frac{x^2}{\sqrt{9-x^2}}dx$ **(b)** $\int \frac{1}{t^2\cdot\sqrt{1+t^2}}dt$

(c) $\int \frac{\sqrt{9-x^2}}{x^2}dx$ **(d)** $\int x^3\cdot\sqrt{9-x^2}\cdot dx$

6. The graphs of the functions $f(x)$, $g(x)$, $r(x)$, $s(x)$, $p(x)$ and $q(x)$ are shown below. You can assume that the domain of each function includes: $0 \le x < \infty$ and that the asymptotic trends shown in this diagram continue beyond the edges of the graph shown.



Assuming that the improper integral $\int_0^\infty f(x)dx$ converges and the improper integral $\int_0^\infty g(x)dx$ diverges, determine whether each of the integrals listed below converges, diverges, or if more information is needed to say for sure.

(a) $\int_0^\infty r(x)dx$ **(b)** $\int_0^\infty s(x)dx$

(c) $\int_0^\infty q(x)dx$ **(d)** $\int_0^1 \frac{f(x)}{g(x)}dx$

(e) $\int_0^\infty p(x)dx$ **(f)** $\int_0^\infty x \cdot q'(x)dx$

7. In this problem your goal is to determine whether each of the following improper integrals converges or diverges. If an integral converges, calculate its value. You should initially try to determine the convergence or divergence of each integral directly using antiderivatives and limits. If you cannot do this for a particular integral you can use a comparison. Note that comparing integrals does not help you to determine the value of a convergent improper integral, however.

(a) $\int_0^4 \frac{1}{\sqrt{16-x^2}}dx$ **(b)** $\int_0^4 \frac{1}{y^2-14}dy$

(c) $\int_2^\infty \frac{1}{x \cdot \ln(x)}dx$ **(d)** $\int_4^\infty \frac{1}{x^2-1}dx$

8. In this problem your goal is to calculate approximate values for definite integrals whose exact values cannot be calculated directly. For each of the definite integrals listed below, approximate it value using (i) the Trapezoid rule, (ii) the Midpoint rule and (iii) Simpson's rule.

(a) $\displaystyle\int_0^3 \frac{1}{1+t^2+t^4}\,dt$ using 6 rectangles.

(b) $\displaystyle\int_4^6 \ln\left(x^3+2\right)dx$ using 10 rectangles.

(c) $\displaystyle\int_0^1 \sqrt{x}\cdot e^{-x}\cdot dx$ using 10 rectangles.

9. The improper integral $\displaystyle\int_1^\infty \frac{1}{x^5\cdot\left(e^{\frac{1}{x}}-1\right)}dx$ arises in physics in connection with black body radiation and Planck's Radiation Law. In this problem your goal will be to decide whether this improper integral converges or diverges.

(a) Find a formula for the tangent line to the curve $y = e^x$ at the point $x = 0$.

(b) Using your answer to (a), explain why: $1 + x \leq e^x$. (Hint: draw a graph showing $y = e^x$ and its tangent line.)

(c) Show that for all real numbers x, $e^{\frac{1}{x}} - 1 > \frac{1}{x}$.

(d) Show that $\displaystyle\int_1^\infty \frac{1}{x^5\cdot\left(e^{\frac{1}{x}}-1\right)}dx$ converges.

10. Your goal in this problem is to evaluate each of the integrals shown below. The technique of integration by parts will probably be very helpful here. Do not use your calculator to evaluate the antiderivatives.

(a) $\displaystyle\int_3^5 x\cdot\cos(x)\cdot dx$

(b) $\displaystyle\int_1^3 t\cdot\ln(t)\cdot dt$

(c) $\displaystyle\int_0^5 \ln(1+t)\cdot dt$

(d) $\displaystyle\int_0^1 u\cdot\sin^{-1}\left(u^2\right)\cdot du$