Unit Test 1 Review Problems – Set A

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

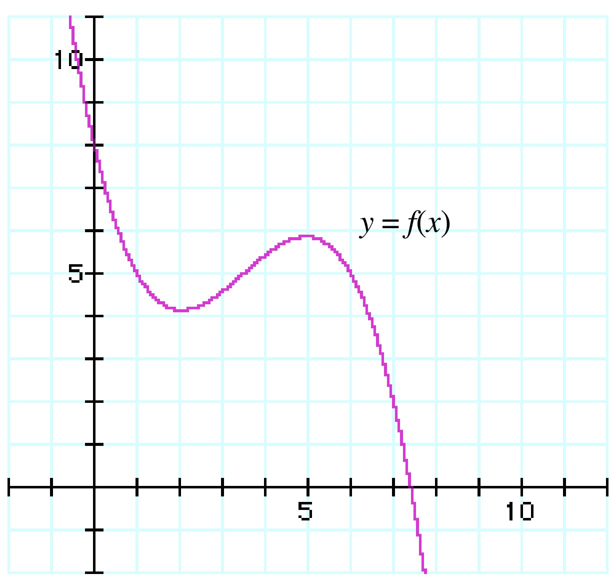
1. In this problem you are required to calculate formulas for each of the anti-derivatives (or indefinite integrals) listed below. Each of these problems can be solved using the technique of u-substitution, although you are welcome to use whatever methods you choose. Your answers should each include an unspecified constant along the lines of "+*C*."

(a) $\int \frac{x \cdot e^{x^2}}{2+e^{x^2}} \cdot dx$ (b) $\int \frac{24 \cdot x^3 + 4}{\left(3 \cdot x^4 + 2 \cdot x + 1\right)^9} \cdot dx$

(c) $\int \frac{1}{2 \cdot \sqrt{x}} \cdot \frac{1}{(1+x)} \cdot dx$ (d) $\int \frac{1}{x \cdot \ln(x)} \cdot dx$

(e) $\int \frac{1}{9+x^2} \cdot dx$ (f) $\int \frac{-2x}{\sqrt{1-x^2}} \cdot dx$

2. In this problem, the function $f(x)$ will always refer to the function defined by the graph shown below.



(a) In terms of area under the curve and the ways that you have learned to approximate such areas, what does the symbolic statement:

$$\sum_{k=0}^{3} f(k\cdot 2)\cdot 2$$

represent?

(b) Sketch the area that is actually represented by the symbolic statement $\sum_{k=0}^{3} f(k\cdot 2)\cdot 2$.

(c) Using the graph given above to find the values of $f(x)$ that you need, find the numerical value of the symbolic statement: $\sum_{k=0}^{3} f(k\cdot 2)\cdot 2$.

(d) The equation of the function $f(x)$ is:

$$y = \tfrac{-1}{7}\cdot(x-1)\cdot(x-3.5)\cdot(x-6)+5.$$

Use the "sum" and "seq" commands on your calculator to approximate the area beneath $y = f(x)$ between $x = 0$ and $x = 8$ with a left hand Riemann sum that includes 100 rectangles.

(e) Using integral notation, write down a symbolic statement that will represent the precise "true" value of the area between the curve $y = f(x)$ and the x-axis from $x = 0$ to $x = 8$.

(f) Use anti-derivatives to find the numerical value of the symbolic statement that you wrote down in Part (e) of this question.

3. In this problem you are required to calculate formulas for each of the anti-derivatives (or indefinite integrals) listed below. Each of these problems can be solved using the technique of integration by parts, although you are welcome to use whatever methods you choose. Your answers should each include an unspecified constant along the lines of "$+C$."

(a) $\int e^{-w}\cdot\cos(2w)\cdot dx$ (b) $\int_{1}^{\sqrt{3}} \arctan\left(\tfrac{1}{\theta}\right)\cdot d\theta$

(c) $\int \sqrt{x}\cdot\ln(x)\cdot dx$ (d) $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} t^3\cdot\cos(t^2)\cdot dt$

(e) $\int (\ln(q))^2\cdot dq$ (f) $\int e^{\sqrt{x}}\cdot dx$

4. In this problem, all that you can assume about the function $f(x)$ and its derivative $f'(x)$ is that they have the specific values listed in the table on the next page, the definite integrals given on the next page, and that the domains of $f(x)$ and $f'(x)$ both include $0 \le x \le 4$.

x	0	2	4
$f(x)$	π	10	$\sqrt{2}$
$f'(x)$	1	-2	4

- $\int_0^2 f(x^2)dx = 13$
- $\int_2^4 f(x)dx = 8$

(a) Find the exact value of the definite integral: $\int_0^2 (f(x))^2 \cdot f'(x)dx$.

(b) Find the exact value of the definite integral: $\int_2^4 x \cdot f'(x)dx$.

(c) Find the exact value of the definite integral: $\int_0^2 2x \cdot f'(x^2)dx$.

(d) Find the exact value of the definite integral: $\int_0^2 x^2 \cdot f'(x^2)dx$.

5. Use trigonometric identities (or any other methods that you know) to evaluate each of the following integrals.

(a) $\int \cos^4(x) \cdot \tan^3(x) \cdot dx$

(b) $\int \sec^6(\theta) \cdot d\theta$

(c) $\int \tan^2(w) \cdot dw$

(d) $\int \sin^6(u) \cdot \cos^3(u) \cdot du$

(e) $\int p \cdot \cos^2(p) \cdot dp$

(f) $\int (1 - \sin(\theta))^2 \cdot d\theta$

6. In this problem your goal is to evaluate each of the integrals given here using either partial fractions, polynomial long division or completing the square. If some other method of integration seems feasible to you, feel free to try it.

(a) $\int \frac{r^2}{r+1} dr$

(b) $\int \frac{1}{x^2 - x + 1} dx$

(c) $\int \frac{10}{(x-1)(x^2+9)} dx$

(d) $\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx$

(e) $\int \frac{x^3 + x}{x - 1} dx$

(f) $\int \frac{2x^3 + 5x}{x^4 + 5x^2 + 4} dx$

7. In this problem your goal is to evaluate each of the integrals given below using the method of trigonometric substitution (or any other valid method that you think will work). You should give your final answers in terms of the variable listed in the problem below (and not leave them expressed in terms of θ). When you convert your integral back to the original variable try to make use of right-angle triangles to do the conversion (as opposed to using acrsin, arctan, etc.).

(a) $\displaystyle\int_0^2 y^3 \cdot \sqrt{y^2+4} \cdot dy$ **(b)** $\displaystyle\int \sqrt{1-4x^2}\, dx$

(c) $\displaystyle\int_{\sqrt{2}}^2 \frac{1}{x^3\sqrt{x^2-1}}\, dx$ **(d)** $\displaystyle\int w \cdot \sqrt{1-w^2}\, dw$

(e) $\displaystyle\int w \cdot \sqrt{1-w^4}\, dw$ **(f)** $\displaystyle\int \frac{p^5}{\sqrt{p^2+2}}\, dp$

8. In this problem, you will approximate the integral: $\displaystyle\int_0^3 e^{\sqrt{x}} \cdot \sin(x) \cdot dx$ using a variety of different methods. For Parts (a)-(e) of this problem, calculate the approximate value of the integral using the method named and the number of rectangles specified. It is fine to use your calculator to evaluate the sums that you get.

NOTE: Make sure your calculator is in RADIAN mode before you start these calculations.

(a) Left hand Riemann sum, 100 rectangles.

(b) Right hand Riemann sum, 200 rectangles.

(c) Midpoint sum, 300 rectangles.

(d) Trapezoid rule, 400 rectangles.

(e) Simpson's rule, 800 rectangles.

9. In this problem your objective is to determine whether each of the following integrals converges or diverges. As you work through these problems, remember to include all of the necessary steps in your calculation. At minimum these would include:

(I) Write the improper integral as a definite integral with a limit.

(II) Use antiderivatives and the techniques of integration you have learned to evaluate the definite integral. One of the limits of integration will normally be *a* or *b*, so your answer should include this.

(III) Take the limit of your antiderivative.

(a) $\displaystyle\int_0^\infty x \cdot e^{-x} \cdot dx$ **(b)** $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\, dx$

(c) $\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$

(d) $\int_0^{1} \frac{1}{e^x}\,dx$

(e) $\int_1^{\infty} \frac{1}{1+x^2}\,dx$

(f) $\int_1^{\infty} \frac{x+1}{\sqrt{x}}\,dx$

10. In this problem your goal is to approximate the value of the integral $\int_0^2 e^{-x^2}\,dx$ using the midpoint and trapezoid rules. It is fine to use your calculator to evaluate the sums in this problem.

(a) The graph of $y = e^{-x^2}$ is shown below for the interval [0, 2]. Based on this graph, is it possible to say which of the midpoint and trapezoid rules will be an overestimate for the integral?

(b) Approximate the integral using the trapezoid rule with 10 rectangles.

(c) Approximate the integral using the midpoint rule with 10 rectangles.

(d) How many rectangles do you have to use if you want to use the trapezoid rule and get an estimate for the integral that is accurate to 0.00001?

(e) How many rectangles do you have to use if you want to use the midpoint rule and get an estimate for the integral that is accurate to 0.00001?

(f) Approximate the integral using Simpson's rule with 20 rectangles.

(g) Estimate the error in your answer to Part (f).