# Outline

1. Approximating alternating sums.
2. Power and Taylor series.
3. Radius of convergence.

- Unit Test 3 : 11/21/08
- Do-over : 12/4/08

## 1. Approximating Alternating Sums.

**Theorem:** Let $\sum_{n=1}^{\infty} (-1)^n \cdot a_n$ be an alternating series.

Suppose $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

$$S = \sum_{n=1}^{\infty} (-1)^n \cdot a_n \text{, sum of series}$$

$$S_N = \sum_{n=1}^{N} (-1)^n \cdot a_n \text{, } N^{th} \text{ partial sum}$$

then:

$$|S - S_N| < a_{N+1}.$$

## Example

Approximate $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{2^n}{n!}$.

accurate to four decimal places.

## Solution

**Need:** Verify that $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{2^n}{n!}$

converges.

Use ratio test. $a_n = \frac{2^n}{n!}$

$$\text{Ratio} = \frac{(-1)^{n+1} \cdot a_{n+1}}{(-1)^n \cdot a_n}$$

$$\text{Ratio} = \frac{(-1)^{n+1} \cdot 2^{n+1}}{(n+1)!} \cdot \frac{1}{(-1)^n} \cdot \frac{n!}{2^n}$$

$$= \frac{-2}{n+1}$$

$$\lim_{n\to\infty} |\text{Ratio}| = \lim_{n\to\infty} \frac{2}{n+1} = 0.$$

Since the limit of the ratio is less than 1, $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{2^n}{n!}$ converges absolutely. (Also conditionally.)

<u>So:</u> We can employ the theorem that says:

$$|S - S_N| < a_{N+1}.$$

**Plan:** Approximate $S$ by $S_N$ for finite value of $N$.

**But:** $N$ needs to big enough so first 4 decimal places of $S_N$ are the same as the first 4 decimal places of $S$.

To do this:

$$\underbrace{|S - S_N|}_{0.00000***\ldots} < a_{N+1} < 0.000005$$

number of decimal places of accuracy we need

one more decimal place is zero.

To find $N$, solve:

$$a_{N+1} = \frac{2^{N+1}}{(N+1)!} < 0.000005$$

Using tables on a calculator to solve this gives $N \geqslant 12$.

If we approximate $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{2^n}{n!}$ by $\sum_{n=1}^{12} (-1)^n \cdot \frac{2^n}{n!}$ we will get a value accurate to 4 decimal places.

Evaluating $\sum_{n=1}^{12} (-1) \cdot \frac{2^n}{n!}$ on a calculator gives:

$$S = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{2^n}{n!} \approx -0.8646.$$

## 2. Power and Taylor Series.

- A power series is a series in which at least some of the terms include whole number powers of a variable.

$$\sum_{n=0}^{\infty} \underbrace{C_n}_{\text{constants}} \cdot (x - \underbrace{a}_{\substack{\text{a number called the center} \\ \text{or anchor of the power series.}}})^n$$

- The Taylor series of $f(x)$ based at $x=a$ ( anchor $=a$) is:

$n^{th}$ derivative of $f(x)$ evaluated at $x=a$.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$$