Outline

1. Sigma notation.
2. Partial sums.
3. Convergence and Divergence.
4. Divergence test: n^{th} Term test for Divergence.

1. Sigma Notation

- A series is a list of numbers added together.
- The sum of a series is the total that list of numbers adds up to.
- Finite series: Series with a finite number of terms added together.

e.g. $\underbrace{1 + \frac{1}{2} + \frac{1}{3}}_{} + \frac{1}{4} + \underbrace{\cdots}_{} + \frac{1}{100}$

enough terms at the beginning to establish the pattern.

Represents all of the terms in between.

finite series always has last number in list written down.

- **Sigma notation** is a shorthand notation for a series.

Last value of k for the series

$$1+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{100} = \sum_{k=1}^{100} \frac{1}{k}$$

Value of k to plug into the formula to give the first term in the series.

Formula that shows how to generate all of the numbers in the series.

- **Infinite** series: A series has infinitely many terms added together.

e.g. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \underbrace{\cdots}$

Infinite series ends with ... to show that the list of numbers just keeps going.

- Infinite series in Σ notation:

Series keeps going

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = \sum_{k=1}^{\infty} \frac{1}{k}$$

- <u>Note:</u> If you have a few terms that don't fit the pattern, you can just add those to the Σ notation.

e.g. $7 + 8 + 9 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

$$= 7 + 8 + 9 + \sum_{k=1}^{\infty} \frac{1}{k}.$$

2. Partial Sums

- A summation formula is a formula that lets you work out the sum of a series without actually adding all the terms together.

e.g. Finite Geometric Series:

$$a + a\cdot r + a\cdot r^2 + \ldots + a\cdot r^{n-1} = \frac{a\cdot\left[1-r^n\right]}{1-r}$$

e.g. Infinite Geometric Series $(-1 < r < 1)$

$$a + a\cdot r + a\cdot r^2 + \ldots = \frac{a}{1-r}$$

- Most series do not have convenient summation formulas. So it is very difficult to know the exact sum of most infinite series.

- However, it's usually possible to determine if the sum of a series is finite or infinite.

- To decide this for an infinite series $\sum_{k=1}^{\infty} a_k$:

① Find a formula for the <u>N^{th} partial sum</u> of the series

$$S_N = a_1 + a_2 + a_3 + \cdots + a_N$$

S_N = first N terms of the series added together.

② Calculate: $\lim_{N \to \infty} S_N$.

③ If $\lim_{N \to \infty} S_N$ is finite then $\sum_{K=1}^{\infty} a_K$ adds up to a finite total.

④ If $\lim_{N \to \infty} S_N$ does not exist or is infinite then $\sum_{K=1}^{\infty} a_K$ does not add up to a finite total.

Example

Does $\sum_{k=0}^{\infty} a \cdot r^k$ [redacted] add up

to a finite total when

$-1 < r < 1$?

Solution

$$S_N = a + a \cdot r + a \cdot r^2 + \ldots + a \cdot r^{N-1}$$

$$S_N = \frac{a \cdot [1 - r^N]}{1 - r}$$

approaches zero

$$\lim_{N \to \infty} S_N = \lim_{N \to \infty} \frac{a \cdot [1 - r^N]}{1 - r}$$

$$= \frac{a \cdot [1 - 0]}{1 - r}$$

a finite number.

Since $\lim_{N\to\infty} S_N = \dfrac{a}{1-r}$, a finite number, the infinite series $\sum_{k=0}^{\infty} a \cdot r^k$ adds up to a finite total.

<u>Note:</u> ① If $\lim_{N\to\infty} S_N$ exists and is finite, this limit also gives the sum of the infinite series.

② A finite series <u>always</u> adds up to a finite total.

3. Convergence and Divergence

- If a series has a finite sum then the series is convergent (or said to converge)

- If a series either fails to have a sum or has an infinite sum then the series is divergent (or is said to diverge).

e.g.

① $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n}+\dots=2$

Convergent series
(Geometric with $a=1$ $r=1/2$).

② $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots=\sum_{k=1}^{\infty}\frac{1}{k}$

Divergent series
(Infinite total.)

③ $$\sum_{k=1}^{\infty}(-1)^k = \underbrace{\underbrace{\underbrace{\underbrace{-1}_{-1}+1}_{0}-1}_{-1}+1}_{0}-1+1-\dots$$

Divergent series.
(Never adds up to a definite value.)