Outline

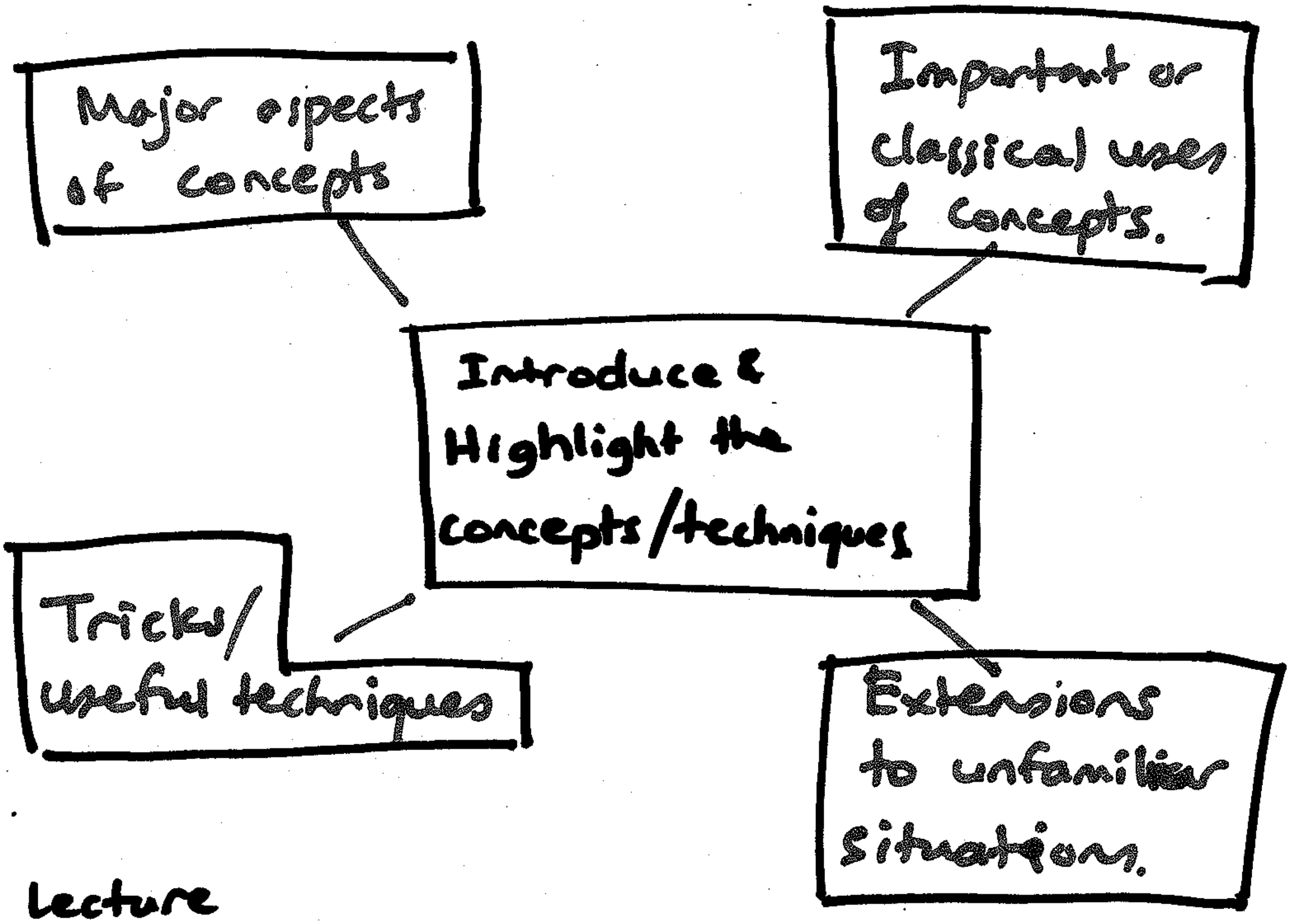
1. Modification Rule.
2. Infinite Series.

Test 2 Do-Over

Thursday October 30

7pm, 8pm, 9pm

2315 Doherty Hall.



Lecture
Recitation/HW.
Quiz

Milestones in Understanding:

1. Vague sense of what's going on - can follow someone else's solution but can't produce solution yourself.

2. Solve problems to some extent - get stuck but can't usually get self unstuck.

3. Solve problems, get self unstuck when difficulties occur. lengthy process.

4. Solve problems, overcome difficulties w/o help and do so efficiently, under all circumstances.

1. Modification Rule

$y_h(x)$ = homogeneous solution

$y_p(x)$ = particular solution.

- Sometimes a term that you want for $y_p(x)$ has already been used in $y_h(x)$.
- Take the term that you want to use in $y_p(x)$ and multiply it by x.

(May have to do this more than once to get a term that is not in $y_h(x)$.)

Example

Find a formula for $y(x)$:

$$y'' - 2y' + y = e^x$$

$$y(0) = 0 \qquad y'(0) = 1.$$

Solution

Step 1: Homogeneous equation.

$$y'' - 2y' + y = 0$$

Char. Eq$\underline{n}$: $r^2 - 2r + 1 = 0$

$$(r-1)(r-1) = 0$$

Roots: $r = 1$ (Real, repeated).

$$y_h(x) = (C_1 + C_2 x) \cdot e^x$$

$$y_h(x) = C_1 e^x + C_2 x e^x$$

Step 2: Particular solution.

$$\left.\begin{array}{l} N(x) = e^x \\ N'(x) = e^x \end{array}\right\} \text{What's important: } e^x.$$

Particular solution: $y_p(x) = \underset{\text{constant.}}{F} \cdot e^x$

$F \cdot e^x$ is no good as a particular solution as $C_1 \cdot e^x$ is a term in $y_h(x)$.

Find value of F by subst. $F \cdot e^x$ into the nonhomogeneous D.E.

$$y'' - 2y' + y = e^x$$

$$\underbrace{Fe^x - 2Fe^x + Fe^x}_{0} = e^x$$

$$0 = e^x$$

So $y_p(x) = F \cdot e^x$ is no good.

- Modification rule:

$$y_p(x) = F \cdot x \cdot e^x$$

This is no good because $C_2 x e^x$ is in $y_h(x)$.

- Modification rule again:

$$y_p(x) = F \cdot x^2 \cdot e^x$$

- Find value of F.

$$y_p'(x) = 2Fxe^x + Fx^2e^x$$

$$y_p''(x) = 2Fe^x + 2Fxe^x + 2Fxe^x + Fx^2e^x$$

Plug $y_p(x) = Fx^2e^x$ into:

$$y_p''(x) - 2y_p'(x) + y_p(x) = e^x$$

$y_p''(x) \quad 2Fe^x + 4Fxe^x + Fx^2e^x$

$y_p'(x) \quad -4Fxe^x \quad -2Fx^2e^x$

$y_p(x) \quad + Fx^2e^x \qquad\qquad = e^x$

$$2Fe^x = e^x$$

$$F = 1/2.$$

Particular solution: $y_p(x) = \frac{1}{2}x^2e^x$.

Step 3: Initial Conditions

$$y(x) = y_h(x) + y_p(x)$$

$$= c_1 e^x + c_2 xe^x + \frac{1}{2}x^2e^x$$

$\underline{y(0)=0:}$ $0 = c_1 + c_2(0) + \frac{1}{2}(0)$

$$\boxed{c_1 = 0}$$

$\underline{y'(0)=1:}$ $1 = c_2 e^0 + c_2(0)e^0$

$+ (0)e^0 + \frac{1}{2}(0^2)e^0$

$$\boxed{c_2 = 1}$$

Final answer:

$$y(x) = x \cdot e^x + \frac{1}{2}x^2 e^x.$$