Outline

1. Nonhomogeneous D.E.'s

2. Method of Undetermined Coefficients.

Next test:

- Friday October 24 During lecture time.
- Practice problems online
- Chapter 7 + additional topics on D.E.s.

1. Nonhomogeneous D.E.'s

Homogeneous: $\underbrace{y'' + y}_{\substack{\text{y and its} \\ \text{derivatives}}} = \underset{\substack{\uparrow \\ \text{zero.} \\ \text{(homogeneous)}}}{0}$

Nonhomogeneous: $y'' + y = \underbrace{0.001 \cdot x^2}_{\substack{\text{Non-zero} \\ N(x).}}$

2. The Method of Undetermined Coefficients

- Good for finding a formula for $y(x)$ when you are given:

- Non homogeneous differential equation (D.E.)
- Two initial conditions.

- Three main steps:
  - **Step 1:** Solve the homogeneous D.E.

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    Create a formula $y_h(x)$.
  - **Step 2:** Create a formula called the *particular solution*.

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    Create a formula $y_p(x)$.
  - **Step:** Put $y_h(x)$, $y_p(x)$ and initial values together to create $y(x)$.

$$y(x) = y_h(x) + y_p(x)$$

Example

Find a formula for $y(x)$

given:

$$y'' + y = 0.001 \cdot x^2$$

$$y(0) = 0 \qquad y'(0) = 1.5$$

Solution

Step 1: Solve the homogeneous equation

Homogeneous: $$y'' + y = 0$$

Char. Eqn $$r^2 + 1 = 0$$

Roots: $$r = \underset{\alpha = 0}{0} + \underset{\beta = 1}{1} \cdot i \qquad r = 0 - 1 \cdot i$$

$$y_h(x) = C_1 \cdot e^{0x} \cdot \cos(1 \cdot x) + C_2 e^{0x} \cdot \sin(1 \cdot x)$$

$$y_h(x) = C_1 \cdot \cos(x) + C_2 \cdot \sin(x).$$

Step 2: Creating a Particular Solution

- Focus on $N(x)$.

$$y'' + y = \underbrace{0.001 \cdot x^2}_{N(x)=0.001x^2.}$$

To find the particular solution:

① Write down $N(x)$ and its derivatives until either the derivatives start repeating or the derivatives reach zero.

	What's important?
$N(x) = 0.001x^2$	x^2
$N'(x) = 0.002x$	x
$N''(x) = 0.002$	1
$N'''(x) = 0$	

$\boxed{\text{STOP}}$

② Multiply each term that was important in ① by an unknown constant and add them together.

Use F, G, H for unknown constants.

$$y_p(x) = F \cdot x^2 + G \cdot x + H \cdot 1.$$

③ Plug $y_p(x)$ back into the nonhomogeneous D.E. to find the numerical values of F, G, H.

$$y_p(x) = Fx^2 + Gx + H$$

$$y_p'(x) = 2Fx + G$$

$$y_p''(x) = 2F$$

$$y'' + y = 0.001x^2$$

$$y_p''(x) + y_p(x) = 0.001x^2$$

$$2F + Fx^2 + Gx + H = 0.001x^2$$

Equate coefficients of powers of x:

$$\underline{x^2:} \quad F = 0.001$$

$$\underline{x^1:} \quad G = 0$$

$$\underline{x^0:} \quad 2F + H = 0$$

<u>So:</u> $F = 0.001$, $G = 0$, $H = -0.002$.

$$y_p(x) = 0.001x^2 + 0x - 0.002$$

$$y_p(x) = 0.001x^2 - 0.002.$$

This is the particular solution.

Step 3: Use Initial Values.

$$y(x) = y_h(x) + y_p(x)$$

$$y(x) = C_1 \cdot \cos(x) + C_2 \cdot \sin(x) + 0.001x^2 - 0.002$$

Now use $y(0) = 0$ and $y'(0) = 1.5$ to evaluate C_1 and C_2.

$y(0) = 0$: $0 = C_1 + C_2 \cdot (0) + 0.001(0) - 0.002$

$$0 = C_1 - 0.002$$

$$\boxed{C_1 = 0.002}$$

$y'(0) = 1.5$: $1.5 = -C_1 \cdot \sin(0) + C_2 \cdot \cos(0) + 0.002 \cdot (0)$

$$\boxed{1.5 = C_2}$$

Final answer:

$$y(x) = 0.002\cos(x) + 1.5\sin(x) + 0.001x^2 - 0.002.$$