

Outline

1. Applications.
2. u-Substitution
3. Integration by
Parts.

1. Application

$$a(t) = \frac{11 - 9.4}{120 - 0} t + 9.4$$
$$= 0.0133t + 9.4$$

Want time "T" for
which :

$$\int_0^T a(t) dt = 5.5$$

$$\left[\frac{0.0133}{2} t^2 + 9.4t \right]_0^T = 5.5$$

$$\frac{0.0133}{2} T^2 + 9.4 T = 5.5$$

$$T = \frac{-9.4 \pm \sqrt{(9.4)^2 - 4\left(\frac{0.0133}{2}\right)(-5.5)}}{0.0133}$$

$$= 0.585$$

or

$$-1410.94$$

2. Integral Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{x} dx = \ln(|x|) + C$$

$$\int \ln(x) dx = x \cdot \ln(|x|) - x + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C, a > 0$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \tan(x) dx = -\ln(|\cos(x)|) + C$$

$$\begin{aligned}\int \frac{1}{1+x^2} dx &= \tan^{-1}(x) + C \\ &= \arctan(x) + C\end{aligned}$$

3. u- Substitution

Pattern:

derivative of
inside function

$$\int_0^2 \frac{3x^2 + 1}{\sqrt{x^3 + x + 1}} dx$$

inside function

① Identify inside function.

$$u = x^3 + x + 1$$

② Calculate du/dx

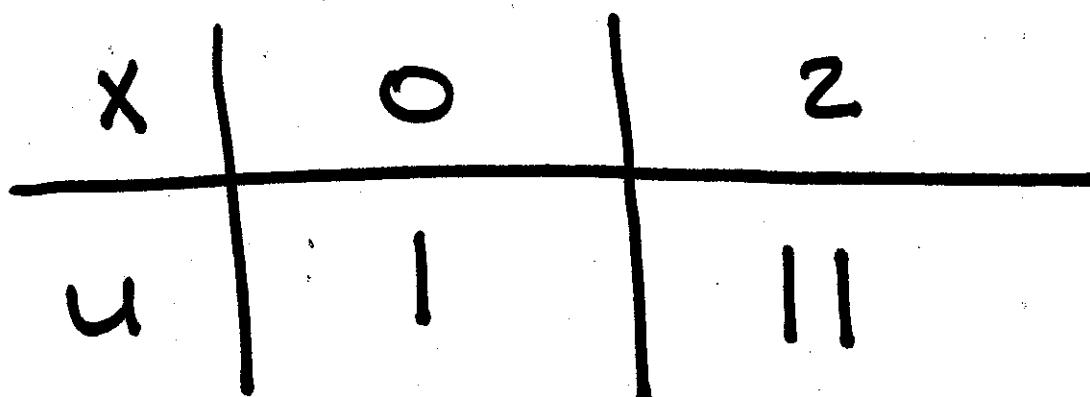
$$\frac{du}{dx} = 3x^2 + 1$$

③ Make dx subject of derivative equation

$$dx = \frac{du}{3x^2 + 1}$$

④ Convert limits of integration

$$u = x^3 + x + 1$$



⑤ Substitute u , dx ,
 limits of integration
 into integral.

$$\begin{aligned}
 & \int_0^2 \frac{3x^2 + 1}{\sqrt{x^3 + x + 1}} dx \\
 &= \int_1^{\pi} \frac{\cancel{3x^2 + 1}}{\sqrt{u}} \frac{du}{\cancel{3x^2 + 1}} \\
 &= \int_1^{\pi} u^{-1/2} du \\
 &= \left[\frac{u^{1/2}}{1/2} \right]_1^{\pi} \\
 &= 2(\sqrt{\pi} - 1)
 \end{aligned}$$

$$\left[2 \sqrt{x^3 + x + 1} + C \right]_0^2$$

$$\int_0^2 u^{-1/2} du$$

4. Integration by Parts

$$\int u \cdot v' dx = u \cdot v - \int u' \cdot v dx$$

$$\int_a^b u \cdot v' dx = [u \cdot v]_a^b - \int_a^b u' \cdot v dx$$

Example

$$\int x \cdot \cos(x) dx$$

$$u = x$$

$$u' = 1$$

$$v' = \cos(x)$$

$$dv = \cos(x) dx$$

$$v = \sin(x)$$

$$u \cdot v - \int u' \cdot v \, dx$$

$$= x \cdot \sin(x) - \int 1 \cdot \sin(x) \, dx$$

$$= x \cdot \sin(x) + \cos(x) + C$$

Example

$$f(0) = 1 \quad f(2) = 4$$

$$\int_0^2 f(x) \, dx = 7$$

What is: $\int_0^2 x \cdot f'(x) \, dx$?

Solution

$$\int_0^2 x \cdot f'(x) dx$$

$\overbrace{\hspace{10em}}$ $\overbrace{\hspace{10em}}$

$u = x$ $v' = f'(x)$
 $u' = 1$ $v = f(x)$

$$= [x \cdot f(x)]_0^2 - \int_0^2 1 \cdot f(x) dx$$

$$= (2)f(2) - (0)f(0) = \int_0^2 f(x) dx$$

$$= (2)(4) - (0)(1) = 7$$

$$= 8 - 7$$

$$= 1$$