# Outline

1. Applications.
2. u-Substitution
3. Integration by Parts.

1. <u>Application</u>

$$a(t) = \frac{11 - 9.4}{120 - 0} t + 9.4$$

$$= 0.0133t + 9.4$$

Want time "T" for which :

$$\int_0^T a(t)\,dt = 5.5$$

$$\left[\frac{0.0133}{2} t^2 + 9.4t\right]_0^T = 5.5$$

$$\frac{0.0133}{2} T^2 + 9.4T = 5.5$$

$$T = \frac{-9.4 \pm \sqrt{(9.4)^2 - 4\left(\frac{0.0133}{2}\right)(-5.5)}}{0.0133}$$

$$= 0.585$$

or

$$-1410.94$$

## 2. Integral Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \,, \; n \neq -1$$

$$\int \frac{1}{x} dx = \ln(|x|) + C$$

$$\int \ln(x) dx = x \cdot \ln(|x|) - x + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C \,, \; a > 0$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \tan(x)\,dx = -\ln\left(|\cos(x)|\right) + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

$$= \arctan(x) + C$$

# 3. u- Substitution

**Pattern:**

derivative of inside function

$$\int_0^2 \frac{3x^2+1}{\sqrt{x^3+x+1}}\,dx$$

inside function

① Identify inside function

$$u = x^3 + x + 1$$

② Calculate $du/dx$

$$\frac{du}{dx} = 3x^2 + 1$$

③ Make $dx$ subject of derivative equation

$$dx = \frac{du}{3x^2+1}$$

④ Convert limits of integration.

$$u = x^3 + x + 1$$

| $x$ | $0$ | $2$ |
|---|---|---|
| $u$ | $1$ | $11$ |

⑤ Substitute u, dx, limits of integration into integral.

$$\int_0^2 \frac{3x^2+1}{\sqrt{x^3+x+1}}\,dx$$

$$= \int_1^{11} \frac{\cancel{3x^2+1}}{\sqrt{u}} \frac{du}{\cancel{3x^2+1}}$$

$$= \int_1^{11} u^{-1/2}\,du$$

$$= \left[\frac{u^{1/2}}{1/2}\right]_1^{11}$$

$$= 2\left(\sqrt{11}-1\right)$$

$$\left[2\sqrt{x^3+x+1}+C\right]_0^2$$

$$\cancel{\int_0^2 u^{-1/2}\,du}$$

# 4. Integration by Parts

$$\int u \cdot v' \, dx = u \cdot v - \int u' \cdot v \, dx$$

$$\int_a^b u \cdot v' \, dx = \left[ u \cdot v \right]_a^b - \int_a^b u' \cdot v \, dx$$

## Example

$$\int x \cdot \cos(x) \, dx$$

$u = x$

$u' = 1$

$v' = \cos(x)$

$dv = \cos(x) \, dx$

$v = \sin(x)$

$$u \cdot v - \int u' \cdot v \, dx$$

$$= x \cdot \sin(x) - \int 1 \cdot \sin(x) dx$$

$$= x \cdot \sin(x) + \cos(x) + C$$

Example

$f(0) = 1$ $f(2) = 4$

$$\int_0^2 f(x) dx = 7$$

What is: $\int_0^2 x \cdot f'(x) dx$ ?

# Solution

$$\int_0^2 x \cdot f'(x)\,dx$$

$u = x$ $\qquad\qquad$ $v' = f'(x)$

$u' = 1$ $\qquad\qquad$ $v = f(x)$

$$= \Big[x \cdot f(x)\Big]_0^2 - \int_0^2 1 \cdot f(x)\,dx$$

$$= (2)f(2) - (0)f(0) - \int_0^2 f(x)\,dx$$

$$= (2)(4) - (0)(1) - 7$$

$$= 8 - 7$$

$$= 1$$