Outline

1. Equilibrium solutions.
2. Symbolic solutions.
3. Separation of variables.
4. Logistic equation.

90 – 100	A
80 – 89	B
70 – 79	C
60 – 69	D

Median: 81.

1. Equilibrium Solutions

- An equilibrium solution is a curve along which the derivative is zero.

Example

$P(t)$ = # people with disease
t = time.

$$\frac{dP}{dt} = -0.0119\,P^2 + 0.348\,P$$

Find equilibrium solutions.

Solution

$\frac{dP}{dt} = 0$. Plug this into

$$\frac{dP}{dt} = -0.0119 P^2 + 0.348P$$

and solve for P.

$$-0.0119P^2 + 0.348P = 0$$

$$P \cdot (-0.0119P + 0.348) = 0$$

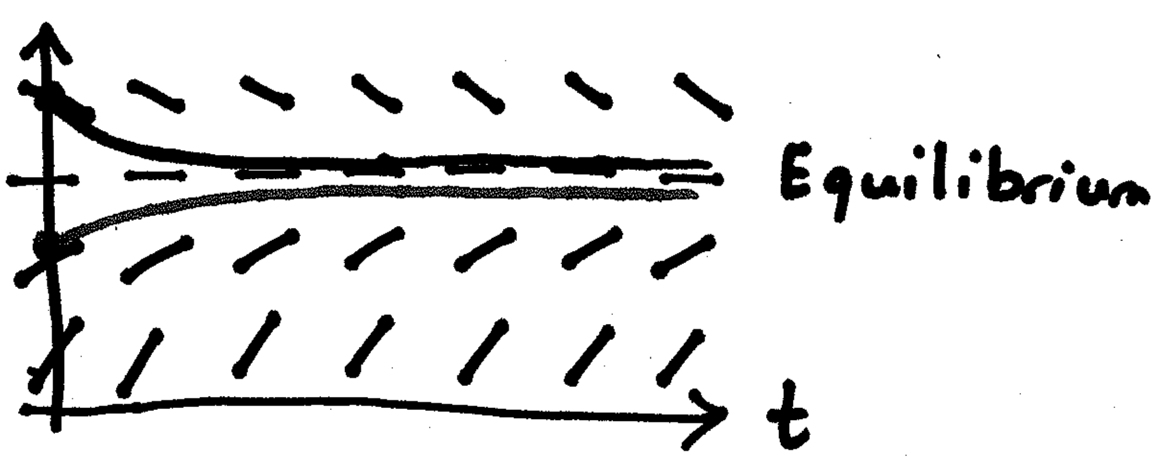
$$P = 0 \qquad P = \frac{0.348}{0.0119} \approx 29.24$$

Equilibrium solutions
(Horizontal lines.)

Three Types of Equilibrium

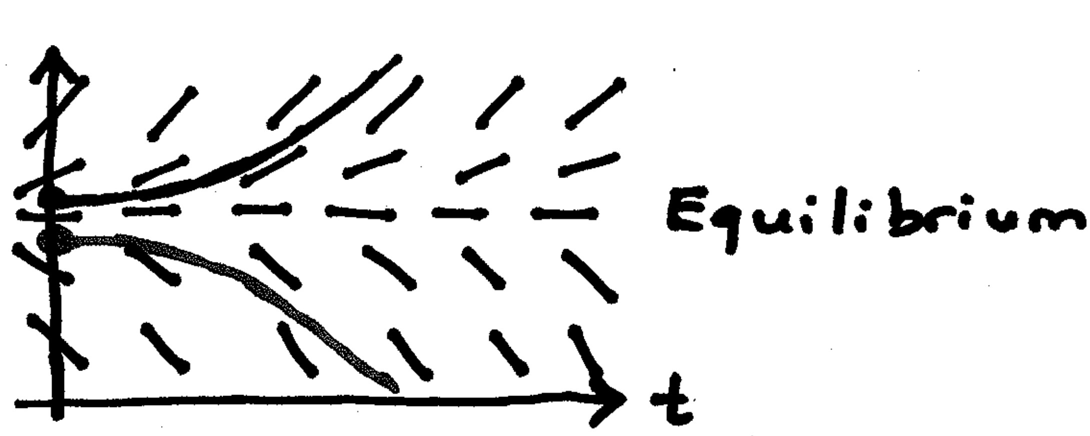
- Key is what do curves do when they get close to the equilibrium?

(a) Stable Equilibrium



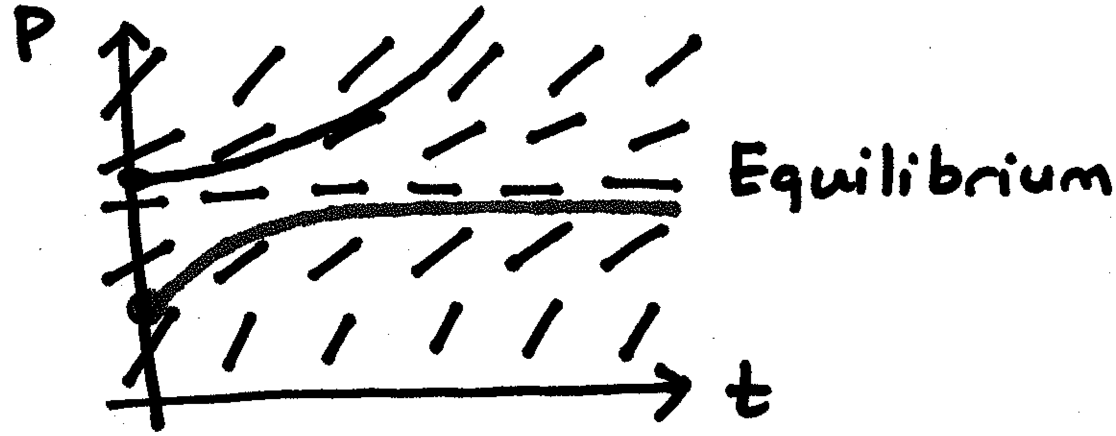
Read left to right:
Curves attracted to equilibrium.

(b) Unstable Equilibrium



Read left to right:
Curves repelled from equilibrium.

(c) Semi-Stable Equilibrium



Read left to right:

Curves are attracted on one side but repelled on the other.

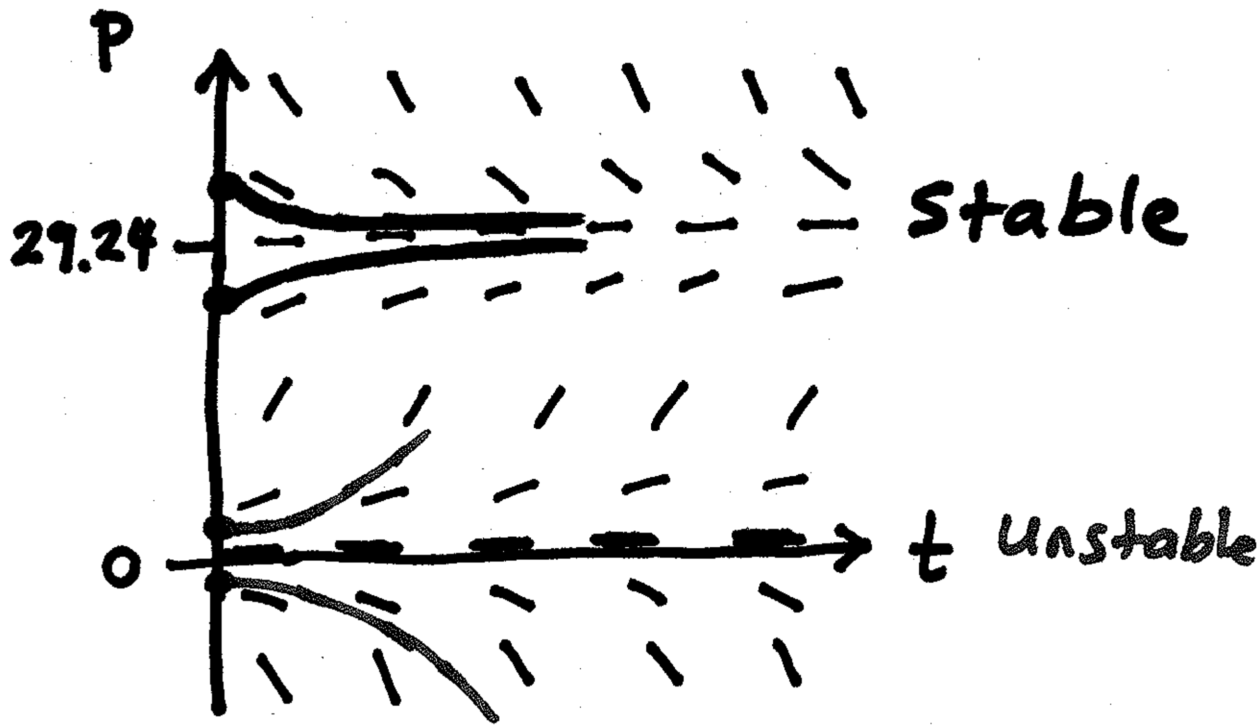
$$\frac{dP}{dt} = (P-3)(P+2)(P-10)^2$$

Example

Classify the equilibrium solutions of :

$$\frac{dP}{dt} = -0.0119\,P^2 + 0.348\,P.$$

Solution



P = 29.24
Stable.

P = 0
unstable.

3. Separation of Variables

- Technique for finding the formula of a function defined by a differential equation.

Example

$$\frac{dW}{dt} = 0.05 \cdot W - 200$$

$$W(0) = 3000.$$

Find the formula for $W(t)$, the function described above.

Solution

- Idea is to rearrange $\frac{dW}{dt}$ to get all W's on one side and all t's on the other.

$$\frac{dW}{dt} = 0.05\,W - 200$$

$$\frac{dW}{dt} = 0.05\left[W - \frac{200}{0.05}\right]$$

$$\frac{dW}{dt} = 0.05\left[W - 4000\right]$$

$$\frac{1}{W-4000}\frac{dW}{dt} = 0.05$$

$$\star \quad \frac{1}{W-4000}\,dW = 0.05\,dt$$

$$\int \frac{1}{W-4000}\, dW = \int 0.05\, dt$$

$$\ln(|W-4000|) = 0.05t + C$$

- Once integration is done, rearrange to make W the subject of the equation.

$$e^{\ln(|W-4000|)} = e^{0.05t + c}$$

$$|W - 4000| = e^{c} \cdot e^{0.05t}$$

Let $A = \pm e^{c}$ constant

$$W - 4000 = A \cdot e^{0.05t}$$

$$W = 4000 + Ae^{0.05t}$$

constant.

- To determine A, use

$$\underbrace{W(0)}_{t=0} = \underbrace{3000}_{w=3000}.$$

$$3000 = 4000 + Ae^{(0.05)(0)}$$

$$3000 = 4000 + A$$

$$-1000 = A.$$

Final answer:

$$W = W(t) = 4000 - 1000e^{0.05t}$$