Outline

1. Euler's Method.
2. Slope fields.
3. Separation of variables.

1. Euler's Method

- **Idea:** Start with:
  1. Derivative
  2. Function value
  3. Time interval / step (Δt)

and approximate other values of the function.

Example

Function: $Q(t)$

Derivative: $\frac{dQ}{dt} = Q - t$

Value: $Q(0) = 3$

Step: $\Delta t = 0.5$

Goal: Approximate value of $Q(2)$.

Current t	Current Q	Deriv. dQ/dt	Rise $Q' \cdot \Delta t$	New Q
0	$\boxed{3}$	$3-0=3$	$(3)(\frac{1}{2})=\boxed{\frac{3}{2}}$	4.5
0.5	4.5	$4.5-0.5=4$	$(4)(\frac{1}{2})=2$	6.5
1.0	6.5	5.5	2.75	9.25
1.5	9.25	7.75	3.875	13.125
2.0	13.125			

So: $Q(2) \approx 13.125$

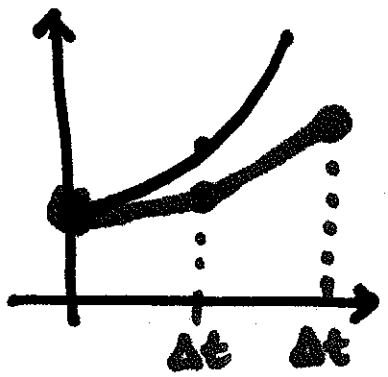
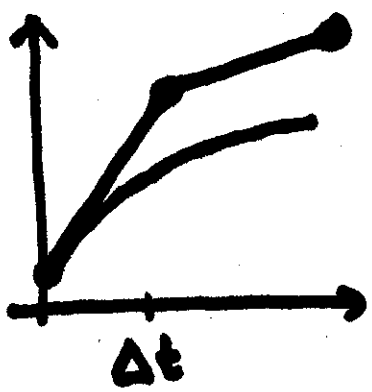
Formula for Euler's Method

$$f(t+\Delta t) \approx \underbrace{f(t)}_{\text{current}} + \underbrace{f'(t)\cdot\Delta t}_{\text{rise}}$$

$$f'(t) \approx \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

Euler's Method and Over/Under-estimates

- Concavity of function is key.

Concavity	Picture	Over/under Estimate
Concave up function		Euler gives an under-estimate of function value.
Concave down function.		Euler gives an over-estimate of the function value.

- You can determine concavity by looking at the derivative column of your Euler method table to see if the derivative increases or decreases.

2. Slope Fields

- Idea: Start with:
  1. Derivative
  2. Function value(s)

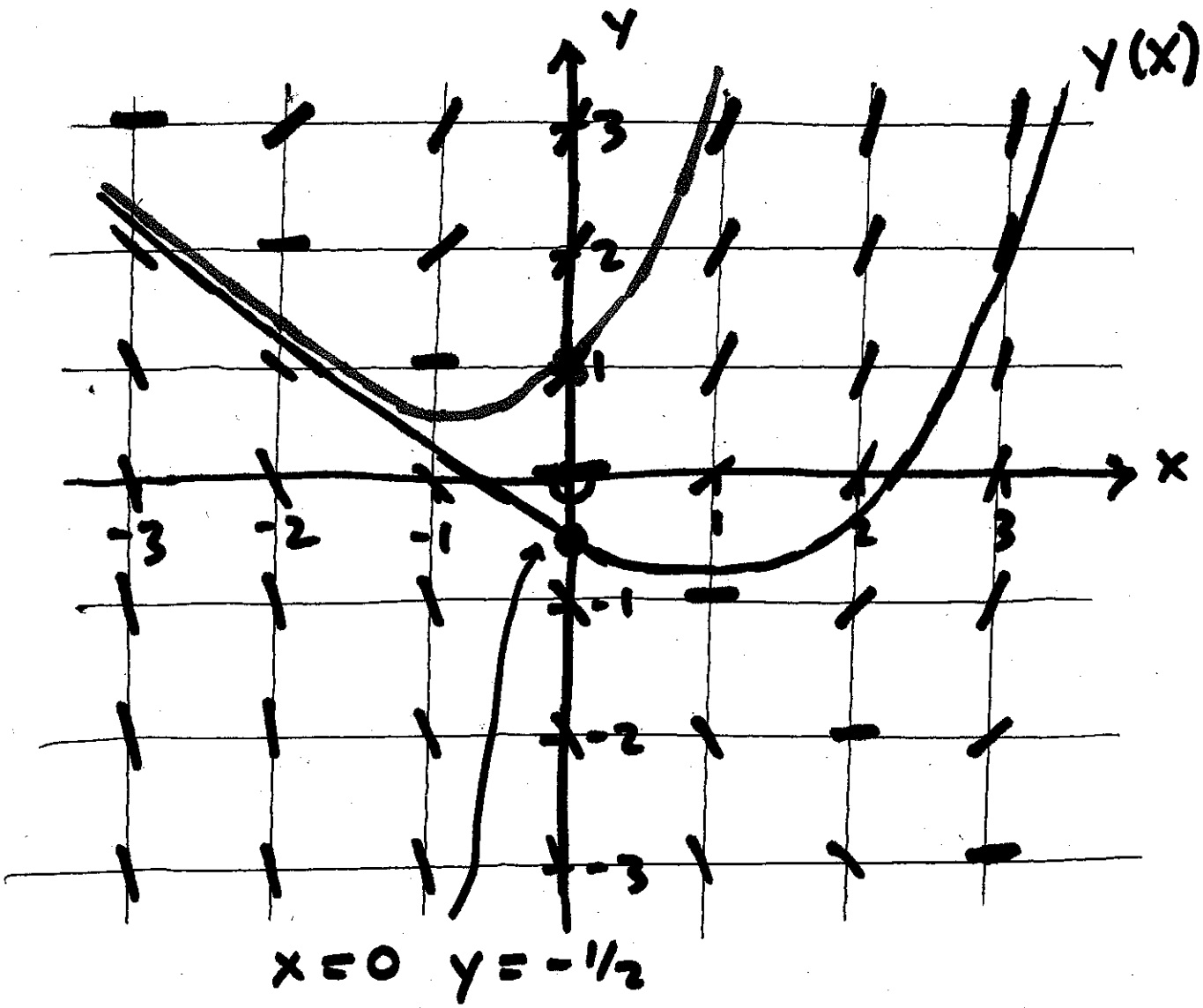
and generate an approximate graph of the function.

Example

$$\frac{dy}{dx} = x + y \qquad y(0) = -1/2$$

Goal: Draw a graph of $y(x)$.

Solution



Example

$$\frac{dy}{dx} = x + y \qquad y(0) = 1.$$

Want a sketch of the graph of $y(x)$.

Solution