

## Outline

1. Euler's Method.
2. Slope fields.
3. Separation of Variables.

# 1. Euler's Method

- Idea: Start with:

- ① Derivative
- ② Function value
- ③ Time interval / step ( $\Delta t$ )

and approximate other values of the function.

## Example

Function:  $Q(t)$

Derivative:  $\frac{dQ}{dt} = Q - t$

Value:  $Q(0) = 3$

Step:  $\Delta t = 0.5$

Goal: Approximate value of  $Q(2)$ .

Current t	Current Q	Deriv. $dQ/dt$	Rise $Q' \cdot \Delta t$	New Q
0	3	$3-0 = 3$	$(3)(\frac{1}{2}) = \frac{3}{2}$	4.5
0.5	4.5	$4.5-0.5 = 4$	$(4)(\frac{1}{3}) = 2$	6.5
1.0	6.5	5.5	2.75	9.25
1.5	9.25	7.75	3.875	13.125
2.0	13.125			

So:  $Q(2) \approx 13.125$

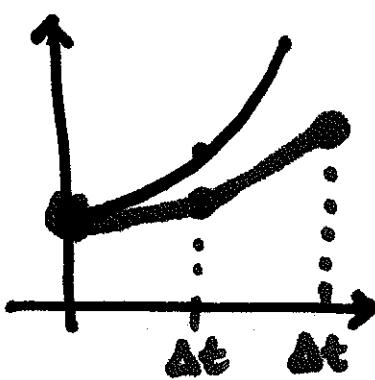
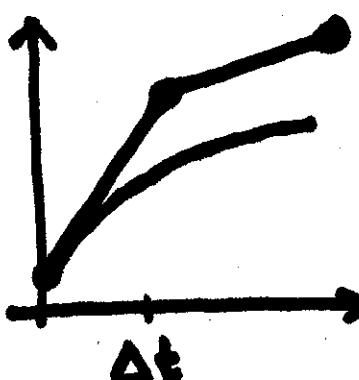
Formula for Euler's Method

$$f(t + \Delta t) \approx \underbrace{f(t)}_{\text{current}} + \underbrace{f'(t)}_{\text{rise}} \cdot \Delta t$$

$$f'(t) \approx \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

# Euler's Method and Over/Under-estimates

- Concavity of function is key.

Concavity	Picture	Over/under Estimate
Concave up function	 A graph showing a concave up function. A secant line connects two points on the curve. The tangent line at the first point lies below the secant line, illustrating that Euler's method provides an under-estimate for concave up functions.	Euler gives an under-estimate of function value.
Concave down function.	 A graph showing a concave down function. A secant line connects two points on the curve. The tangent line at the first point lies above the secant line, illustrating that Euler's method provides an over-estimate for concave down functions.	Euler gives an over-estimate of the function value.

- You can determine concavity by looking at the derivative column of your Euler method table to see if the derivative increases or decreases.

## 2. Slope Fields

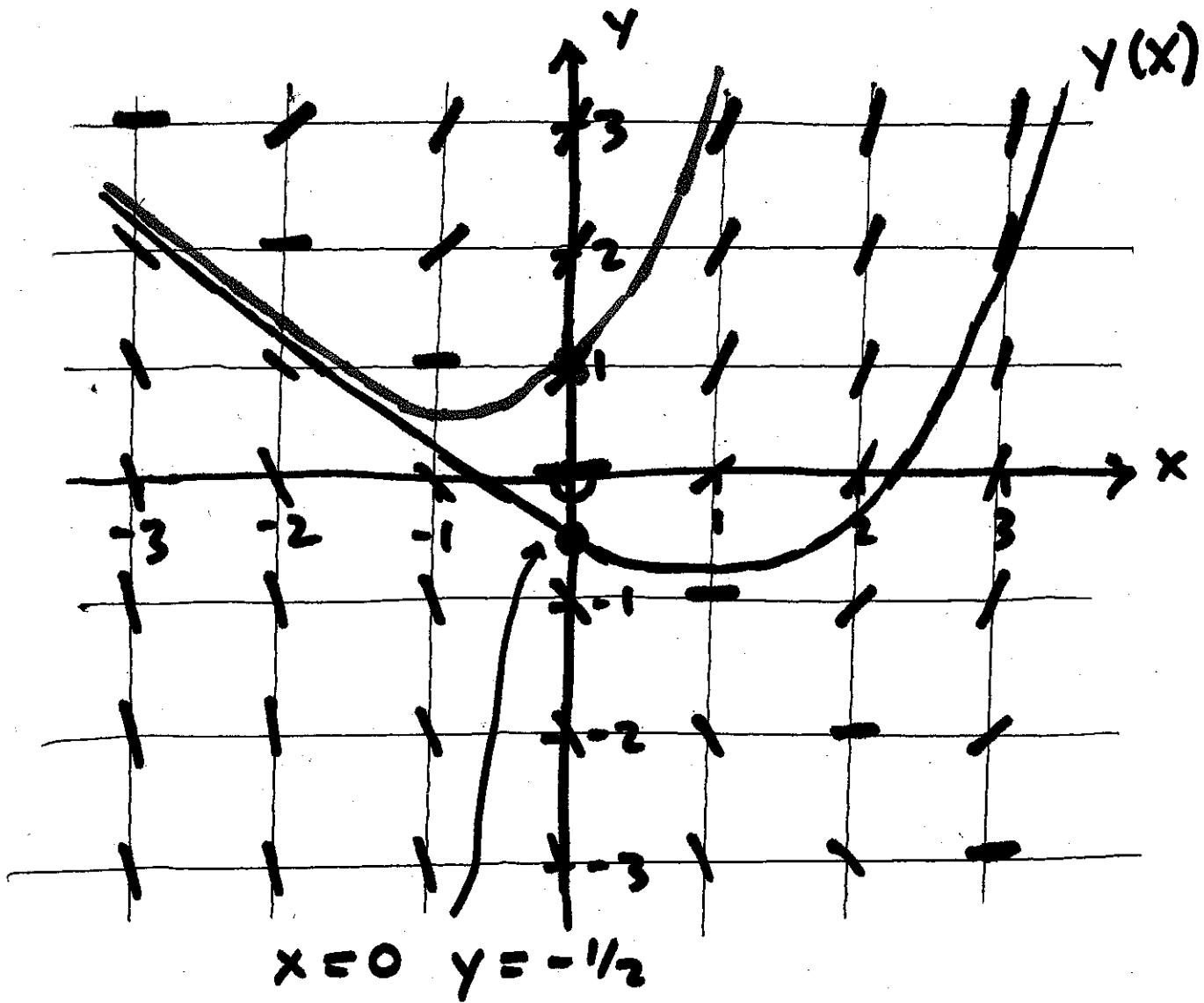
- Idea: Start with:
  - ① Derivative
  - ② Function value(s)and generate an approximate graph of the function.

## Example

$$\frac{dy}{dx} = x + y \quad y(0) = -\frac{1}{2}$$

Goal: Draw a graph of  $y(x)$ .

## Solution



# Example

$$\frac{dy}{dx} = x + y \quad y(0) = 1.$$

Want a sketch of the graph of  $y(x)$ .

# Solution

