# Outline

1. Center of mass.

2. Calculating work.

3. Euler's method.

# 1. Center of Mass

- Center of mass is the balance point of an object.
- The x-coordinate of the center of mass is:

$$\bar{x} = \frac{\int_a^b x \cdot \begin{pmatrix} \text{The mass between} \\ x \text{ and } x + dx \end{pmatrix}}{\int_a^b \begin{pmatrix} \text{The mass between} \\ x \text{ and } dx \end{pmatrix}}$$

- When density $\delta(x)$ has units like $g/cm$ ← one distance unit

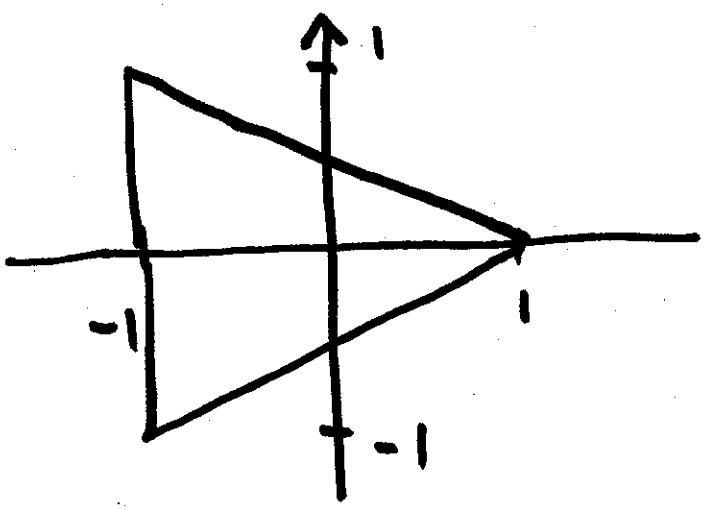
$$\bar{x} = \frac{\int_a^b x \cdot \delta(x) \cdot dx}{\int_a^b \delta(x) \cdot dx}$$

## Example

$$\delta(x) = 1 - x \quad g/cm^2 .$$

two space dimensions.

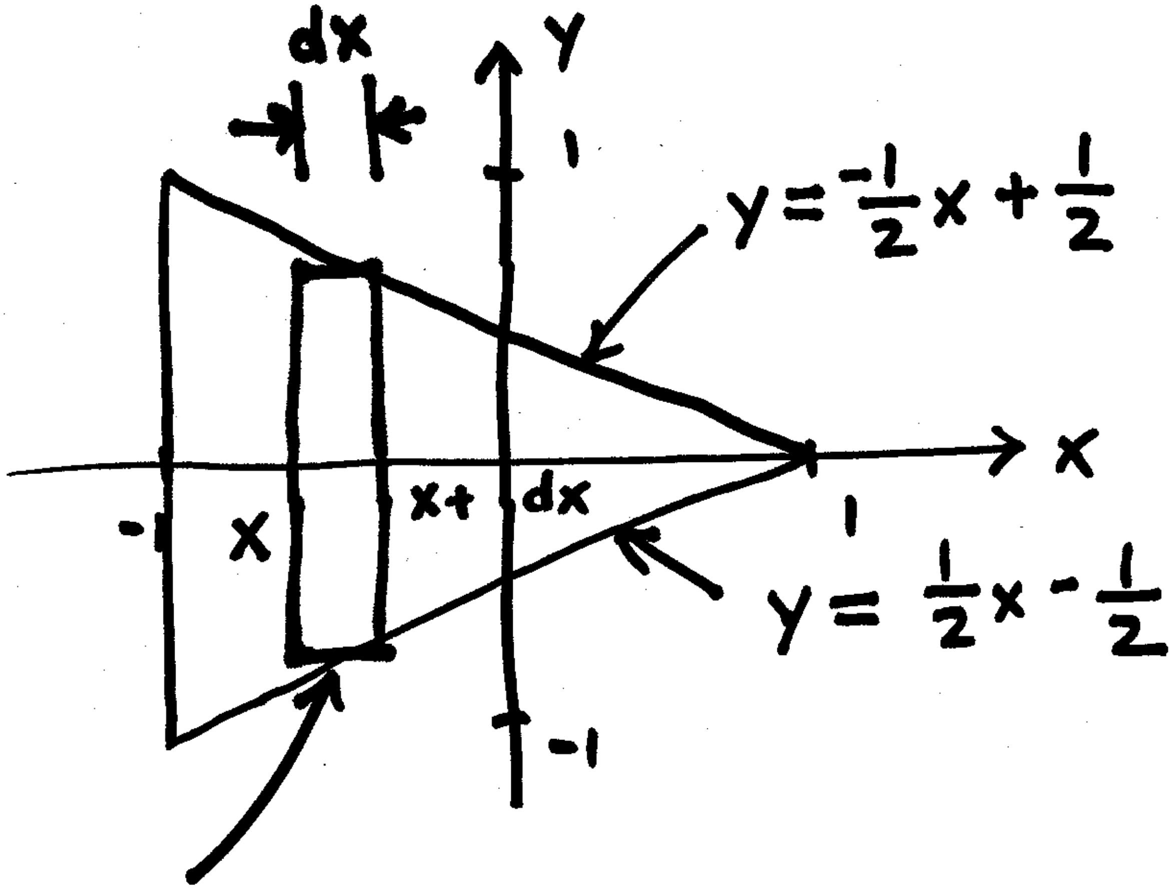
Object:




Where is the x-coordinate of the center of mass?

## Solution

First work out mass between $x$ and $x+dx$.




want mass of this rectangle.

$$\text{Mass of rectangle} = \overset{g}{(\text{density})}\overset{g/cm^2}{}\overset{cm^2}{(\text{Area})}$$

$$= (1-x)\left(\frac{-1}{2}x+\frac{1}{2}-\left(\frac{1}{2}x-\frac{1}{2}\right)\right)\cdot dx$$

$$= (1-x)(-x+1)\,dx$$

Top of Center of Mass

$$= \int_{-1}^{1} x\cdot(1-x)(-x+1)\,dx$$

$$= \int_{-1}^{1} x - 2x^2 + x^3\,dx$$

$$= -4/3$$

Denominator of center of mass $= \int_{-1}^{1} (1-x)(-x+1)\,dx$

$$= \int_{-1}^{1} 1 - 2x + x^2\,dx$$

$$= 8/3$$

x-coord of Center of mass $= \dfrac{-4/3}{8/3} = -1/2.$

## 2. Work

- Measure of energy expended when a force moves an object.

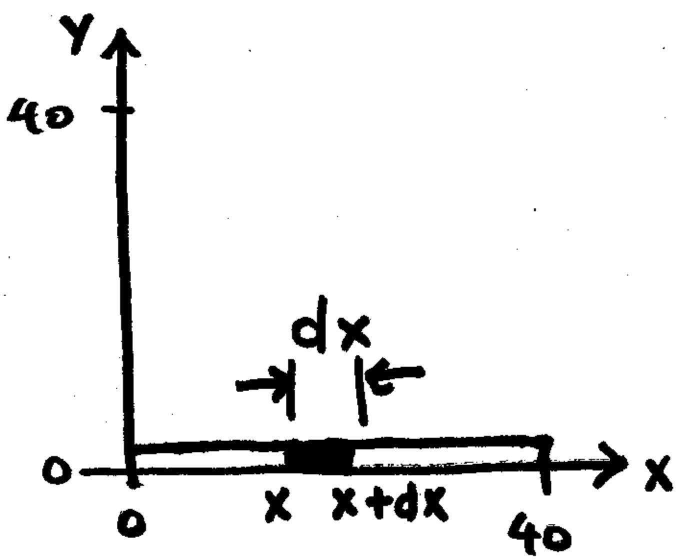
- $\text{Work} = (\text{Force}) \cdot (\text{Distance moved})$

distance parallel to the direction of the force.

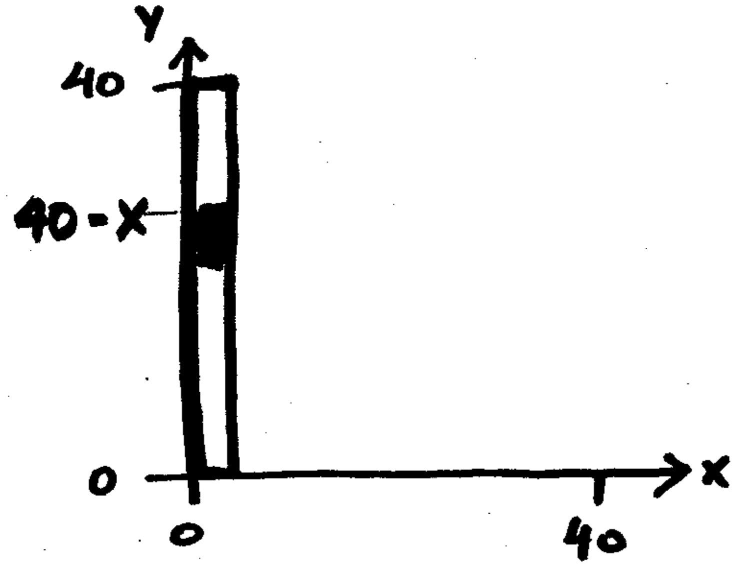
## Example

A 40 m chain (mass of 1 m of chain is 9 kg) is lying flat on the ground. The chain is winched up so it is hanging vertically in the air with only the tip still touching the ground. How much work is done?

Before




After




How much work to lift the piece of chain between x and x + dx ?

$$\text{Force} = \begin{pmatrix}\text{gravitational} \\ \text{acceleration}\end{pmatrix}(\text{mass})$$

$$= (9.8)(9\ \text{kg/m})(\text{length})$$

$$= (9.8)(9)\cdot dx$$

$$\text{Distance} = 40 - x$$

$$\text{Work} = (40-x)\cdot(9.8)(9)\,dx$$

$$\text{Total Work} = \int_0^{40} (40-x)(9.8)(9)\,dx$$

$$= 70,560 \text{ N}\cdot\text{m or J.}$$

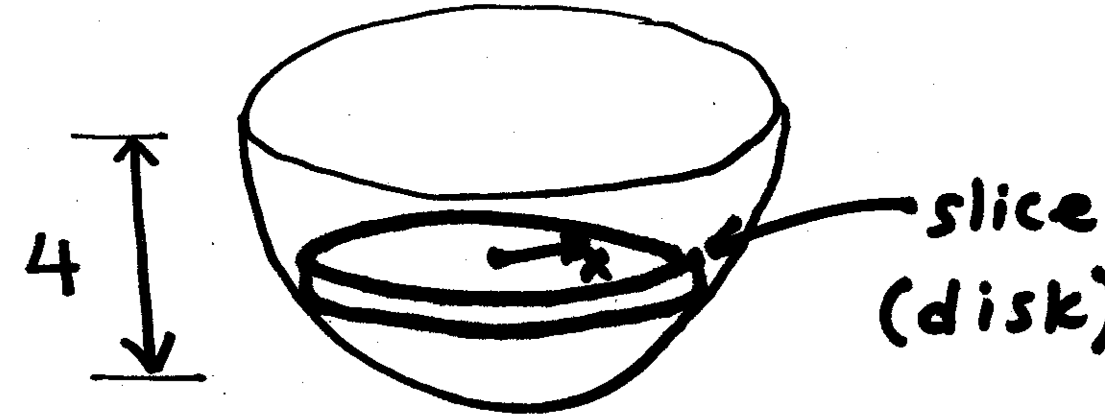
<u>Example</u>

A pond is full of toxic waste ( density = 1100 $kg/m^3$).

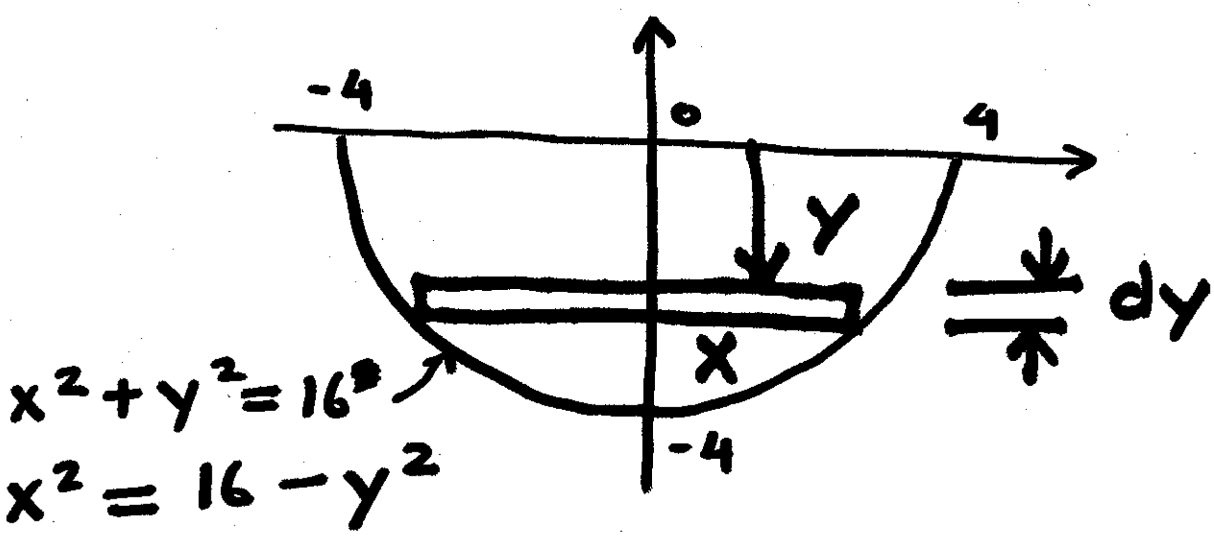
The pond is a hemisphere with a radius of 4m.

How much work to pump all of the toxic waste out of the pond ?

# Solution




Work to remove this slice:




Distance moved $= y$

Force on slice $= (g)$ (mass of slice)

$= (9.8)$ (density) (volume of slice)

$= (9.8)(1100)(\pi \cdot x^2 \cdot dy)$

$= (9.8)(1100)(\pi(16-y^2) \cdot dy)$

Work for slice = (distance) (force)

$$= y \cdot (9.8)(1100)(\pi)(16-y^2)\,dy$$

Total Work $= \displaystyle\int_{-4}^{0} y(9.8)(1100)(\pi)(16-y^2)\,dy$

$$= 2167447.604$$

N·m or J.