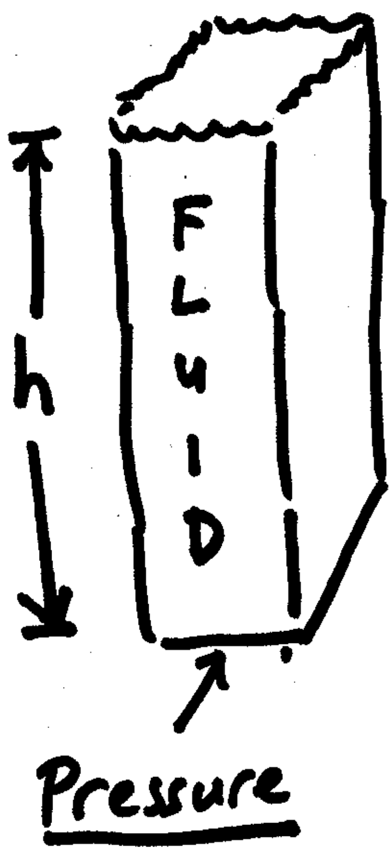
Outline

1. Hydrostatic pressure & force
2. Arc length.
3. Center of mass.

1. Hydrostatic Pressure

- Pressure created by the weight of a still liquid.



Density $= \delta$

Gravitational Acceleration $= g$

Depth $= h$

$$\boxed{P = \delta \cdot g \cdot h}$$

Pressure, P

- At depth h, pressure $P = \delta g h$ is exerted in all directions.

- Second important formula:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\boxed{\text{Force} = (\text{Pressure})(\text{Area})}$$

good when pressure constant.

<u>Example</u>

The wreck of the Titanic is at a depth of about 3790 m.

Density of seawater is 1027 kg.

Circular porthole has its center at a depth of 3790 m and radius 0.5 m.

(a) Calculate pressure at the center of porthole.

(b) Calculate force exerted on porthole if it is horizontal.

(c) Calculate force exerted on porthole if it is vertical.

Solution

(a) $\text{Pressure} = (1027)(9.8)(3790)$

$$= 38144834 \ N/m^2$$

(b). Horizontal Porthole = Constant pressure.

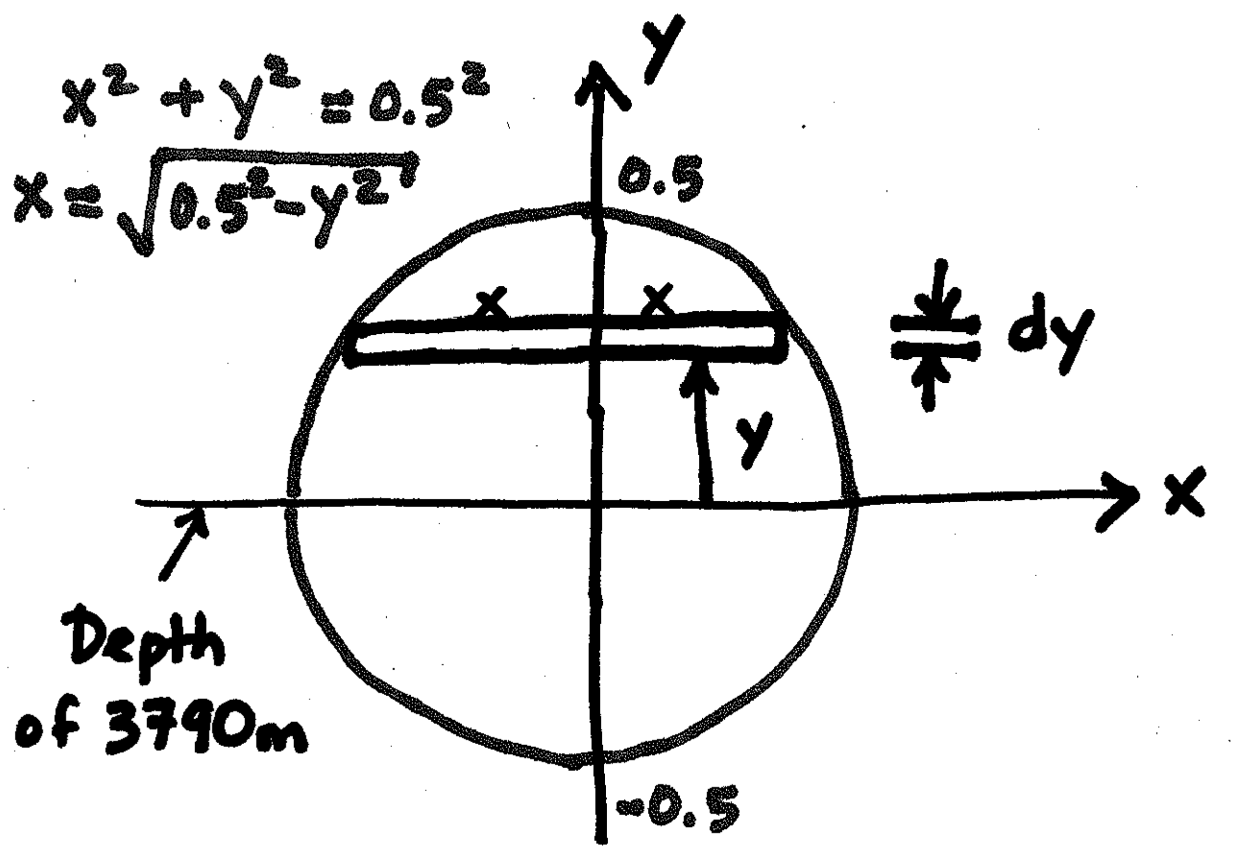
$$\text{Force} = \left(\begin{array}{c}\text{constant}\\ \text{pressure}\end{array}\right)(\text{area})$$

$$= (38144834)(\pi(0.5)^2)$$

$$= 29958882.57 \ N$$

(c) Vertical porthole = pressure varies as you go from top to bottom of the porthole.

y = vertical distance from center of porthole.



Pressure on strip $= (1027)(9.8)(3790 - y)$

Area of strip $= 2x \cdot dy$

$$= 2\sqrt{0.5^2 - y^2} \cdot dy$$

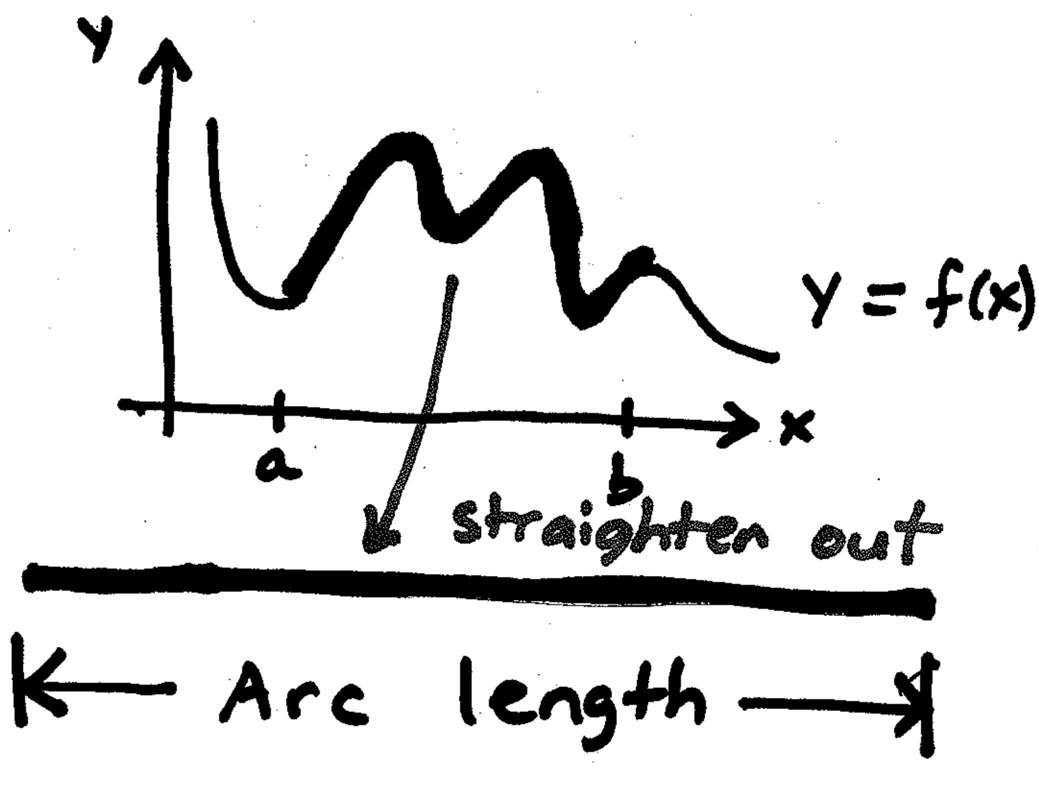
Force on strip $= (1027)(9.8)(3790-y)\, 2\sqrt{0.5^2 - y^2} \cdot dy$

$$\text{Total force} = \int_{-0.5}^{0.5} (1027)(9.8)(3790-y)(2)\sqrt{0.5^2-y^2} \cdot dy$$

$$= 29958882.57 \text{ N.}$$

2. Arc Length

- Length you get from straightening a curve and measuring it.



$$\text{Arc length} = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

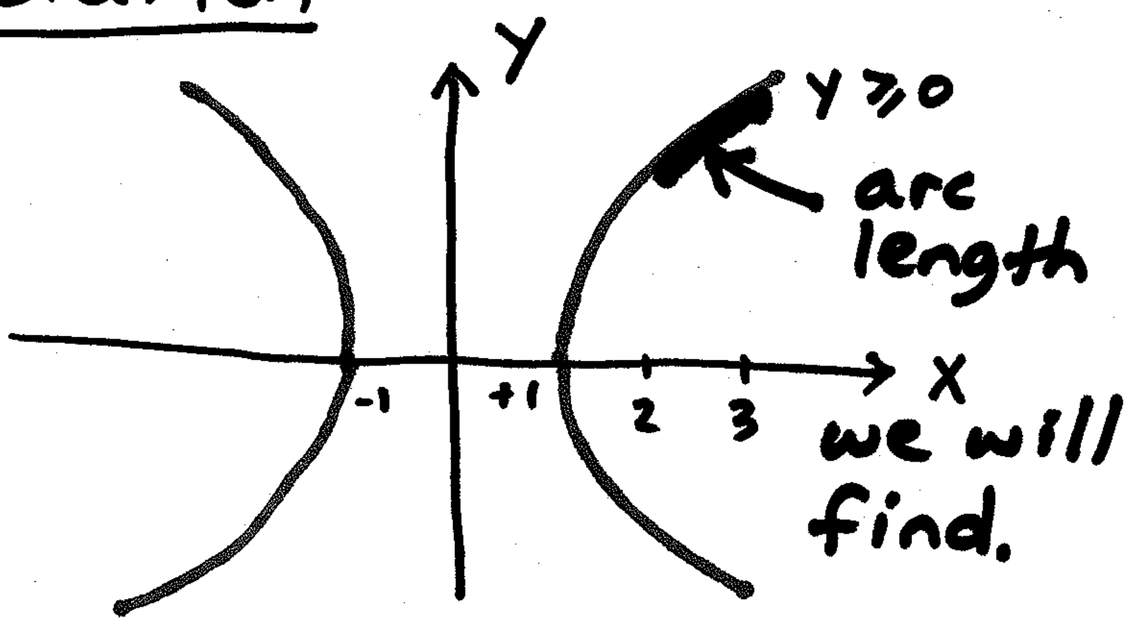
Example

Find the length of the hyperbolic arc:

$$x^2 - y^2 = 1$$

between $x = 2$ and $x = 3$ with $y \geqslant 0$.

Solution



$$x^2 - y^2 = 1$$

$$y = \sqrt{x^2 - 1} = f(x)$$

$$f'(x) = \frac{x}{\sqrt{x^2-1}}$$

$$1 + [f'(x)]^2 = 1 + \frac{x^2}{x^2-1}$$

$$= \frac{2x^2-1}{x^2-1}$$

Arc length $= \int_2^3 \sqrt{\frac{2x^2-1}{x^2-1}}\,dx = 1.48$