

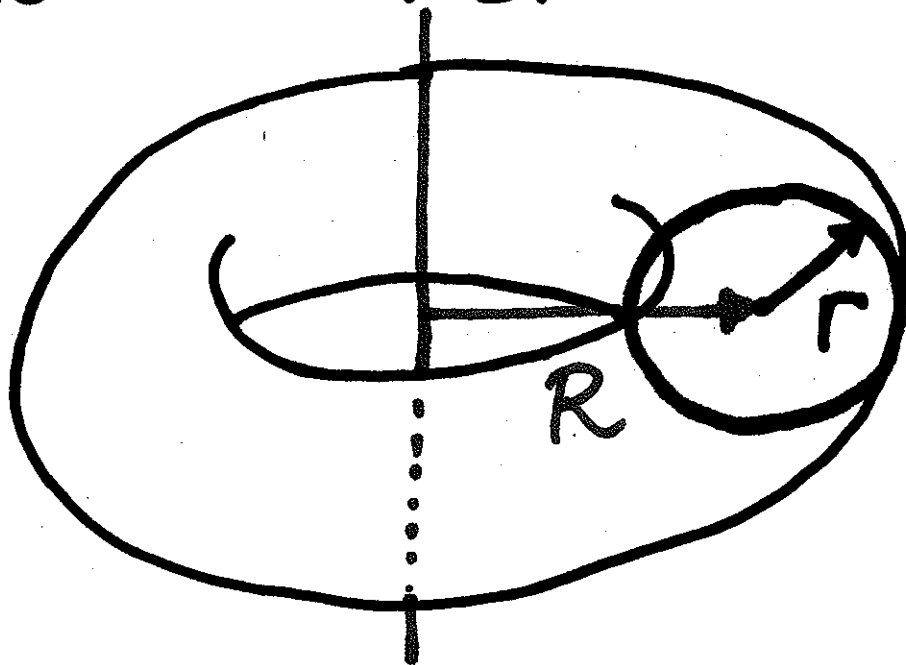
Outline

1. Volume by Shells.
2. Mass and density.
3. Center of mass.

1. Volumes by Shells.

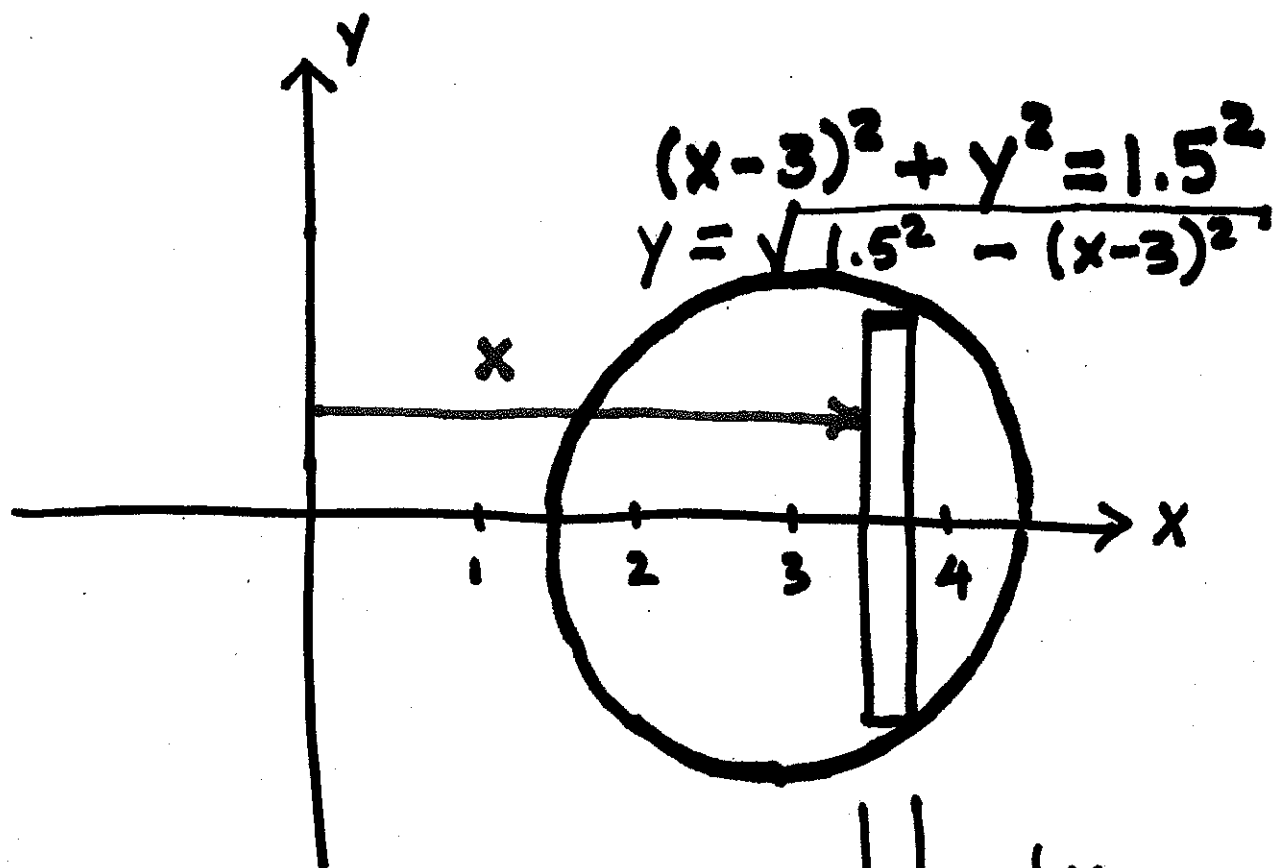
Example

Find the volume of a solid torus with $R=3$ and $r=1.5$.



Axis of rotation

Small circle is revolved around the axis of rotation.



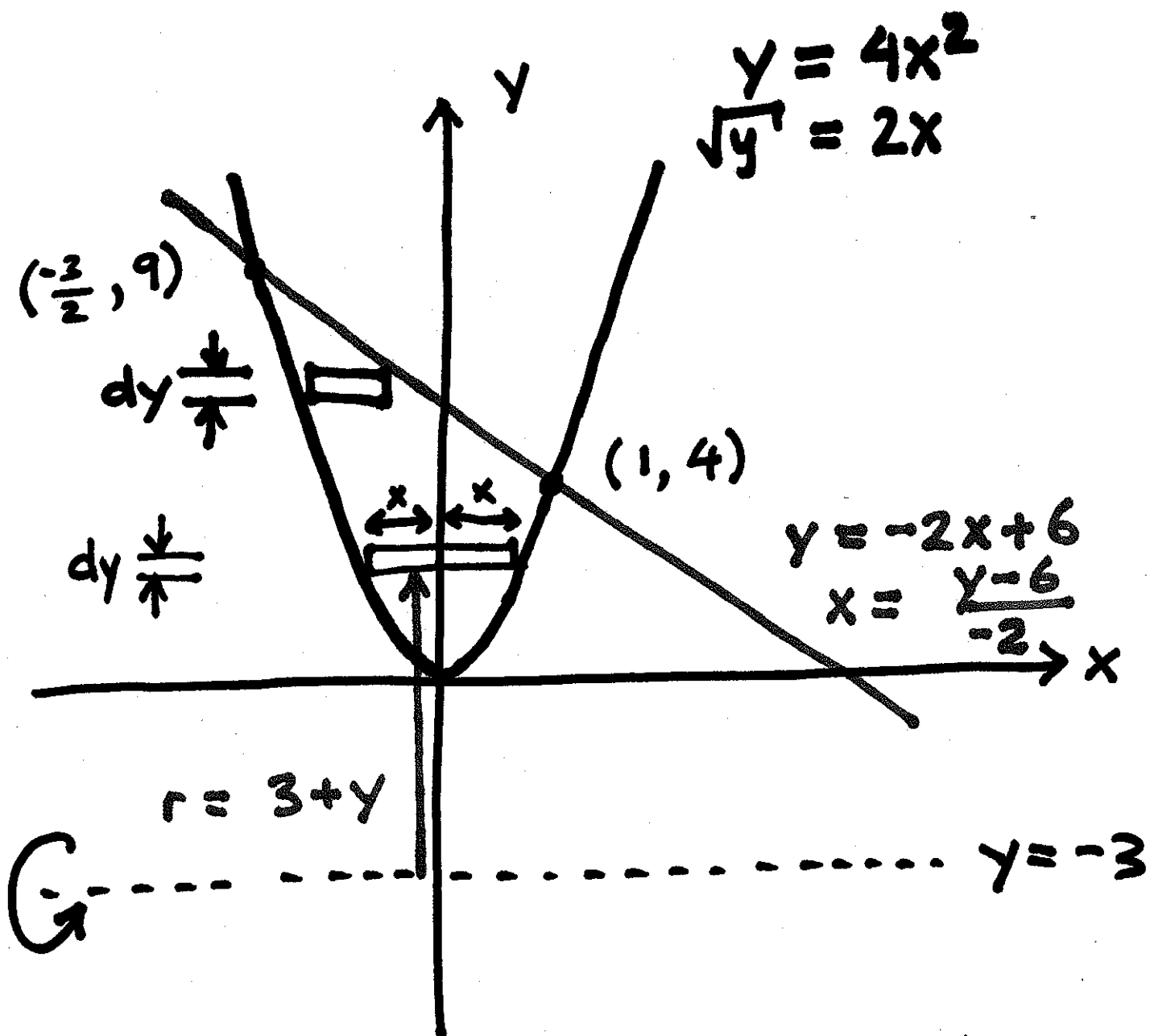
Volume of

$$\text{shell} = 2\pi \cdot x \cdot 2 \cdot \sqrt{1.5^2 - (x-3)^2} \cdot dx$$

Total volume

$$= \int_{1.5}^{4.5} 2\pi x \cdot 2 \cdot \sqrt{1.5^2 - (x-3)^2} \cdot dx$$

$$= 133.24 \text{ cm}^3$$



Orange rectangle's shell volume

$$= 2\pi(3+y)\sqrt{y} \cdot dy$$

Brown rectangle's shell volume

$$= 2\pi(3+y) \cdot \left(\frac{y-6}{-2} - \frac{-\sqrt{y}}{2} \right) \cdot dy$$

$$\begin{aligned} &\text{Total volume} \\ &= \int_0^4 2\pi(3+y)\sqrt{y} \, dy \\ &+ \int_4^9 2\pi(3+y) \left(\frac{y-6}{-2} - \sqrt{\frac{y}{4}} \right) dy \end{aligned}$$

2. Calculating Mass

- Units of density function are important.

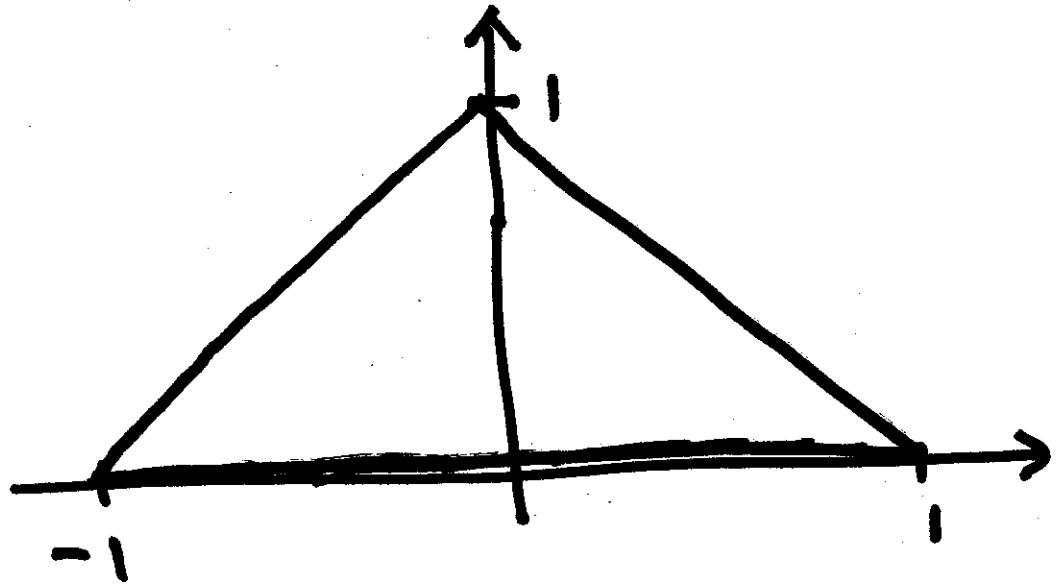
▷ g/cm³: Slice up the object into small volumes that have approximately constant density.

▷ g/cm^2 : Slice up the object into small areas that have approximately constant density.

▷ g/cm : Slice up the object into small lengths that have approximately constant density.

Example

2-D object:



$$\text{Density} = \delta(x) = 1 + x \text{ g/cm}^2$$

If density is constant, can work out mass by:

$$\text{density} = \frac{\text{mass}}{\text{area}}$$

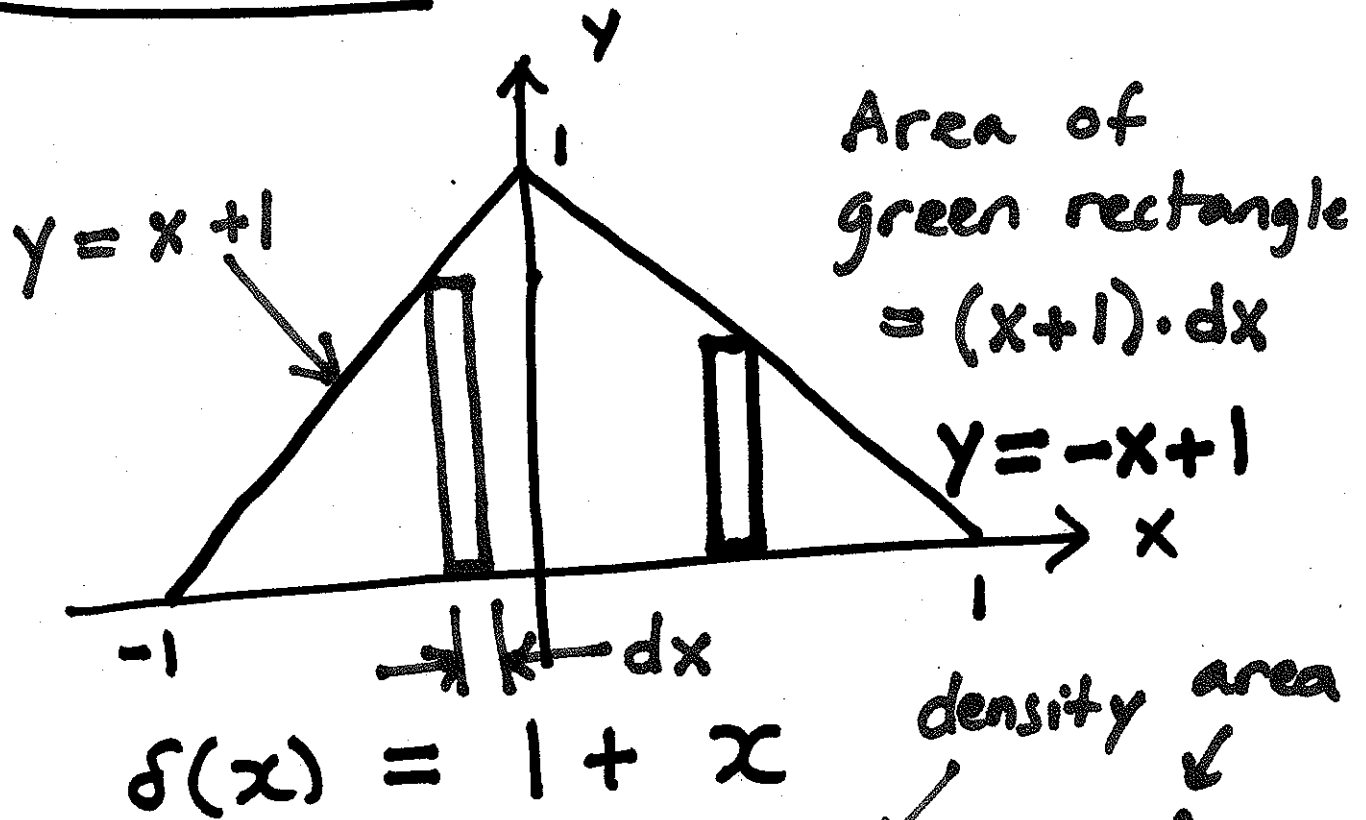
$$\text{mass} = (\text{density}) \cdot (\text{area})$$

When density isn't constant, strategy is to break up

the object into pieces of approximately constant density, so:

$(\text{density}) \cdot (\text{area}) = \text{mass}$
applies on each.

Solution



$$\text{Mass of green rectangle} = (1+x) \cdot (x+1) \cdot dx$$

$$\text{Mass of red rectangle} = (1+x) \cdot (-x+1) \cdot dx$$

Total mass =

$$\int_{-1}^0 (1+x) \cdot (x+1) \cdot dx \quad \text{green mass}$$

$$+ \int_0^1 (1+x) \cdot (-x+1) \cdot dx \quad \text{red mass}$$

$$\begin{array}{l} \text{Area of} \\ \text{Eq. } \Delta \end{array} = \frac{\sqrt{3}}{4} L^2$$