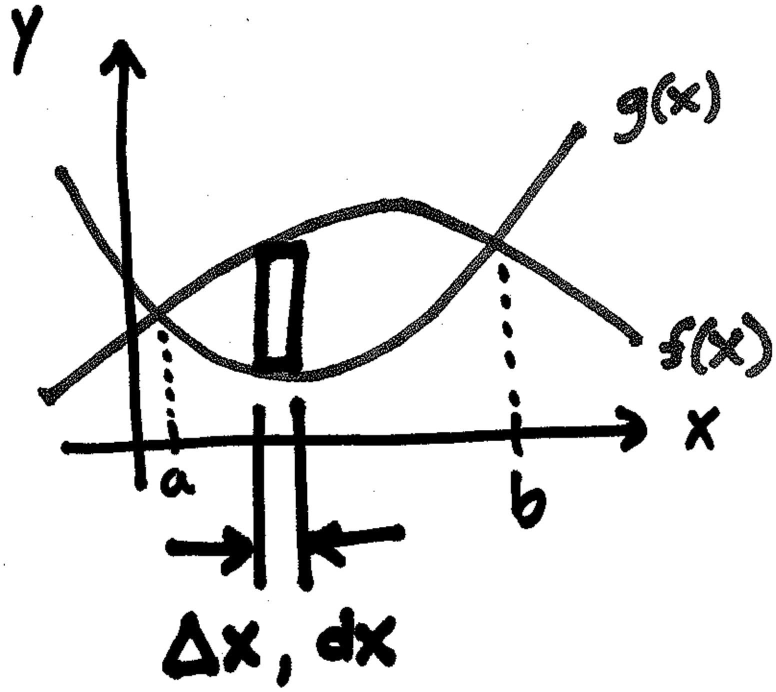
Outline

1. Sideways area.
2. More on disks — Washers.
3. Volumes that don't involve rotation.
4. Volumes by shells.

1. Sideways Area

- **Have:**



Slice area into vertical rectangles. width of Δx or dx

$$\text{Total area} = \int_a^b f(x) - g(x)\, dx$$

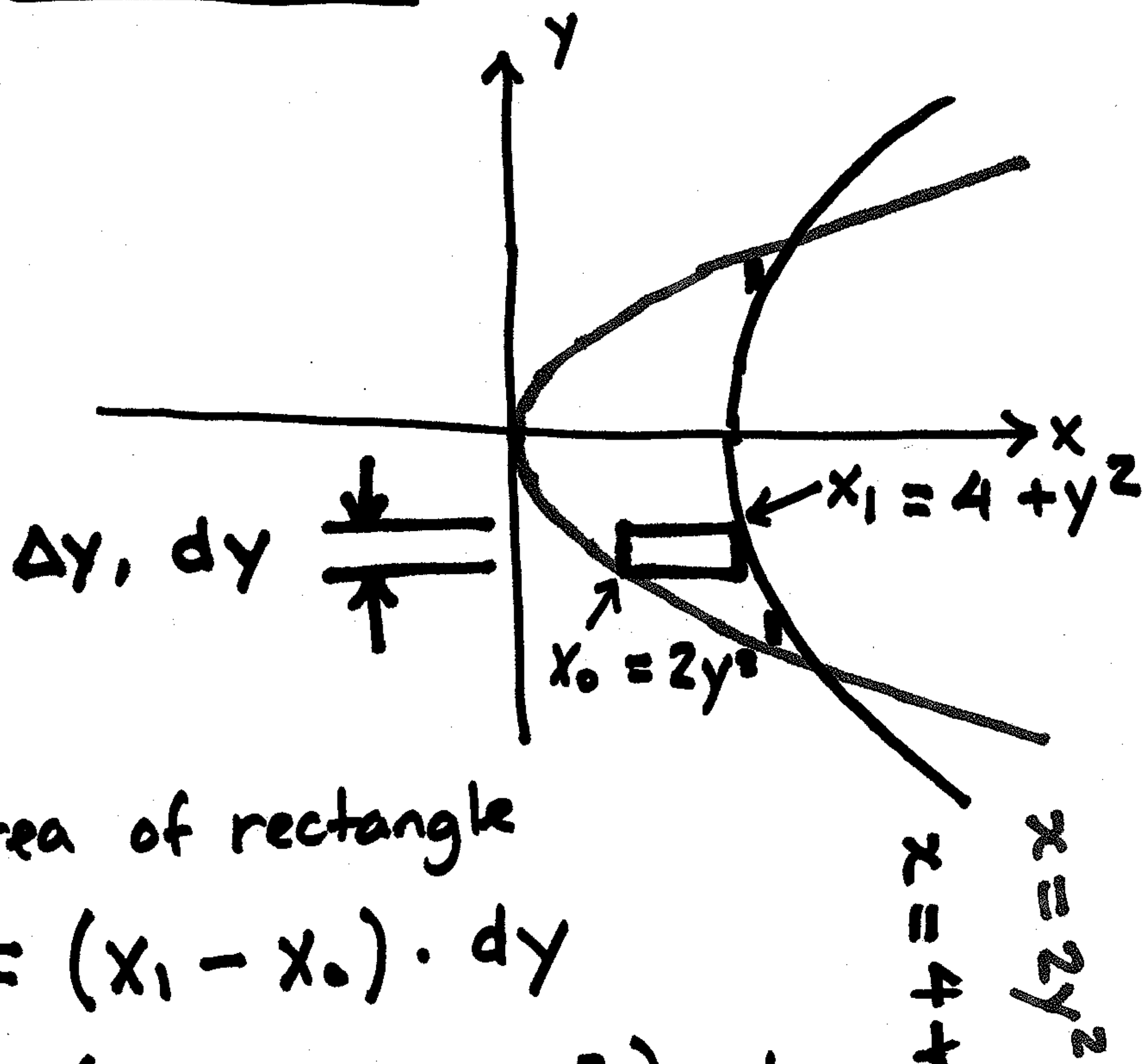
- Sometimes it's easier to slice the area up into horizontal rectangles to give a "dy" integral.

Example

Find area bounded by:

$x = 2y^2 \qquad x = 4 + y^2$

Solution.



Area of rectangle

$= (x_1 - x_0) \cdot dy$

$= (4 + y^2 - 2y^2) \cdot dy$

Total area =

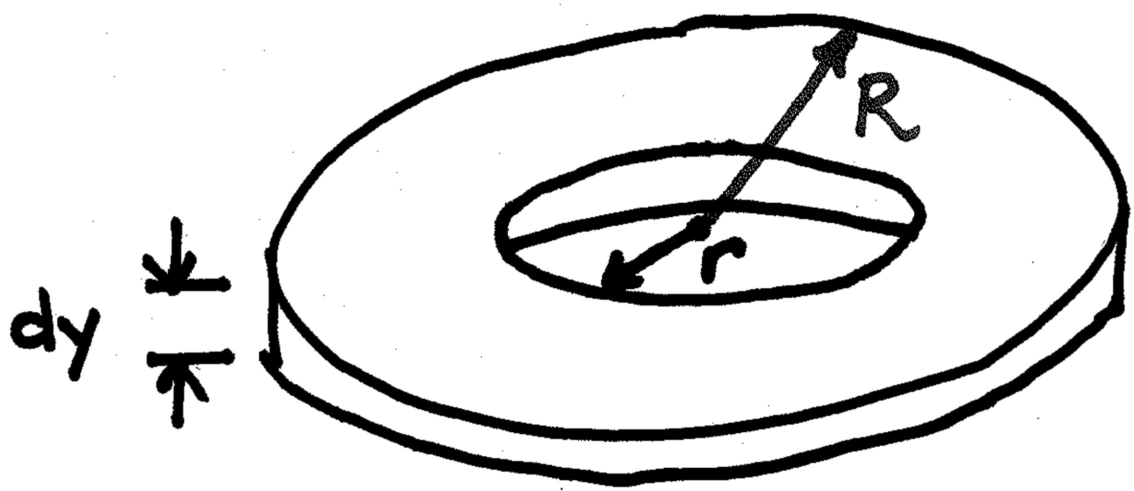
$$\int_{-2}^{2} (4+y^2-2y^2)\,dy = \frac{32}{3}.$$

Limits of integration are y-coordinates of intersection points:

$$4+y^2 = 2y^2$$
$$4 = y^2$$
$$\pm 2 = y$$

2. Volumes with Hollow Centers – Washers

- Volume is sliced into pieces that resemble:



Volume of slice $= (\pi \cdot R^2 - \pi \cdot r^2) \cdot dy$

- Useful for a volume of revolution that has an empty or hollow middle.

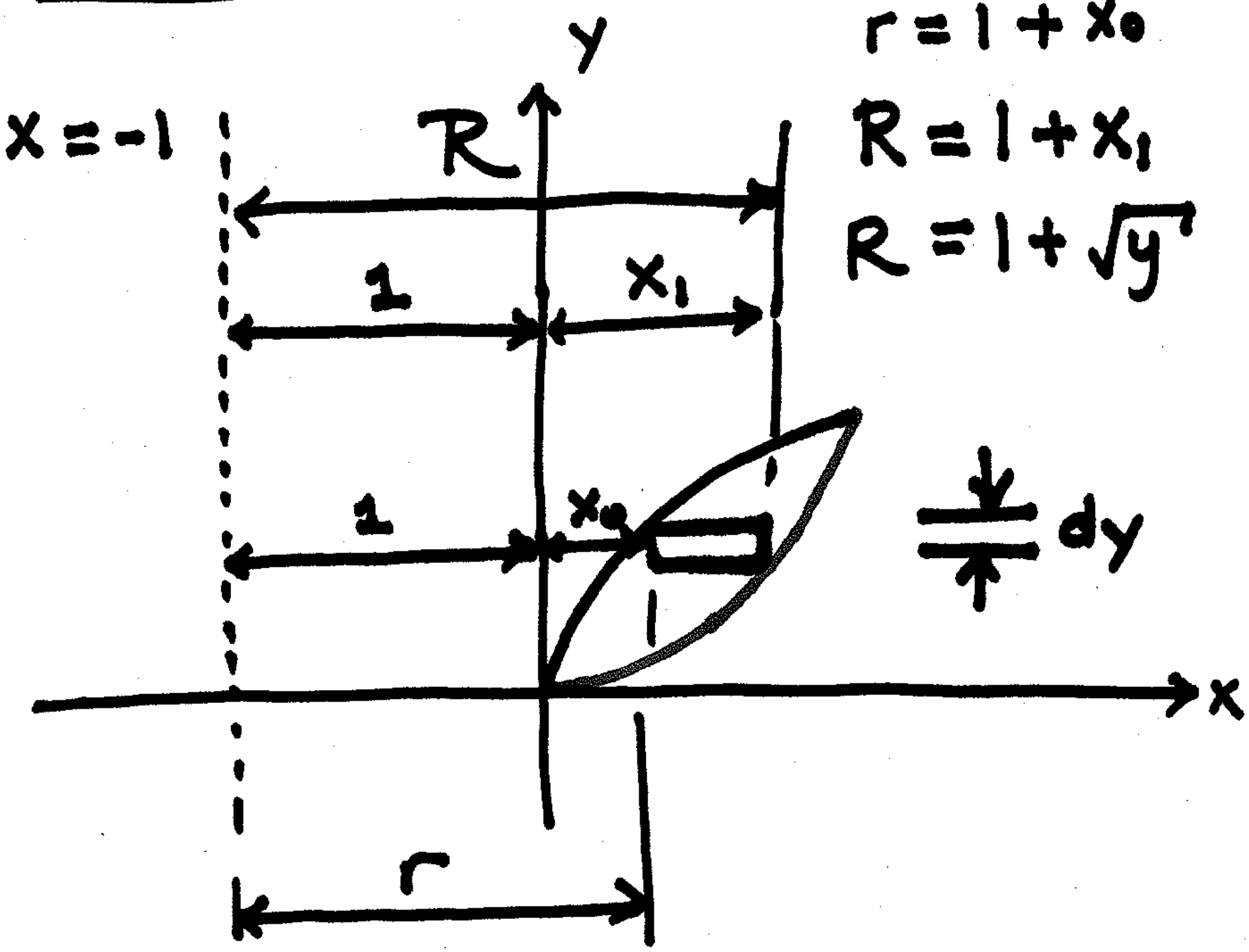
Example

Find volume created when the area bounded by:

$$X = y^2 \qquad\qquad y = x^2$$
$$\sqrt{x} = y \qquad\qquad y = x^2$$

is revolved around $X = -1$.

Solution



$r = 1 + y^2$

$r = 1 + x_0$

$R = 1 + x_1$

$R = 1 + \sqrt{y}$

Volume of slice

$$= (\pi R^2 - \pi r^2) \cdot dy$$

$$= \left(\pi \cdot (1+x_1)^2 - \pi (1+x_0)^2\right) \cdot dy$$

$$= \left(\pi \cdot (1+\sqrt{y})^2 - \pi (1+y^2)^2\right) \cdot dy$$

Total volume

$$= \int_0^1 \left(\pi(1+\sqrt{y})^2 - \pi(1+y^2)^2\right) \cdot dy$$

3. Volumes (but not of Revolution)

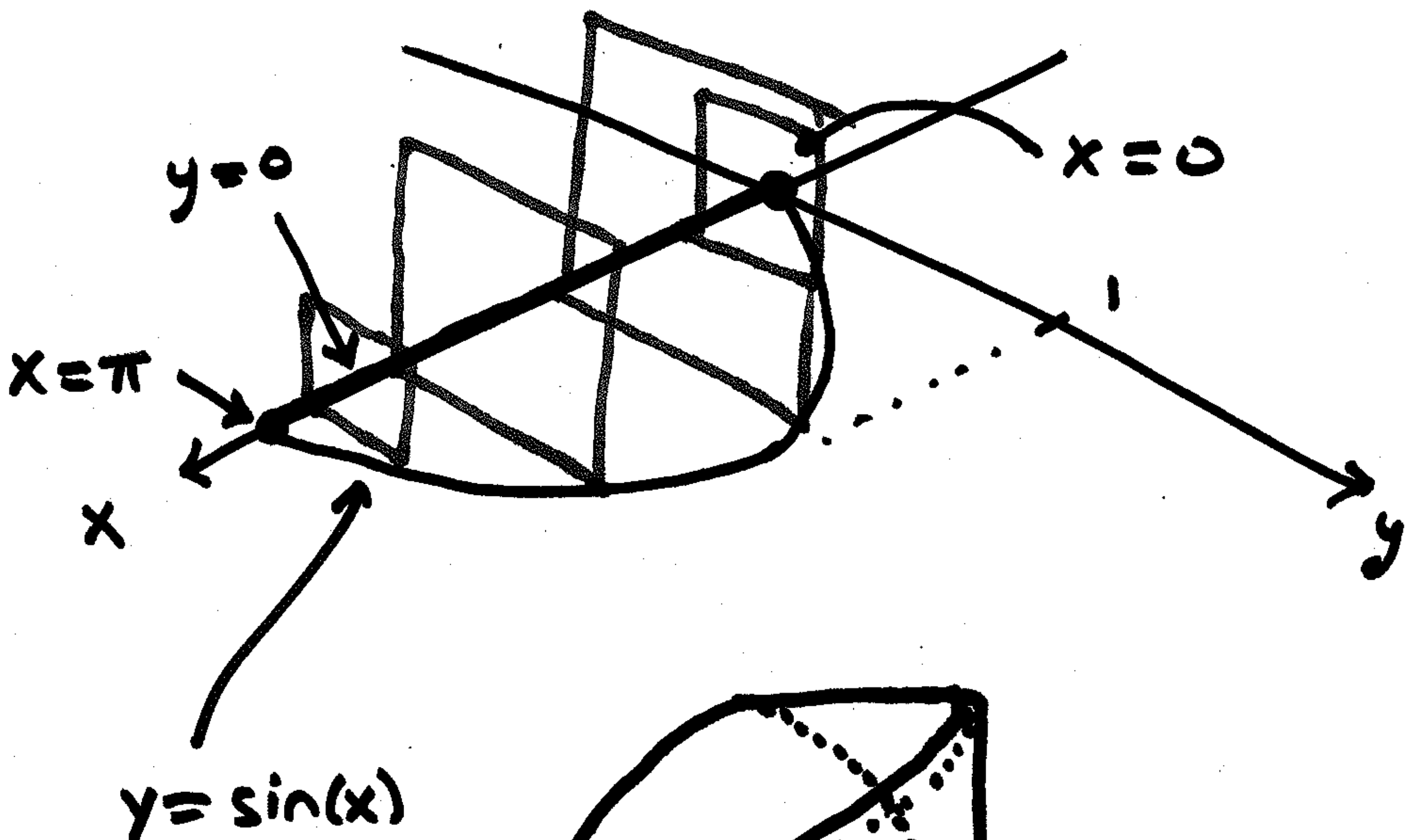
- Typical information:
  - Base of the volume (area in xy plane)
  - Cross-section of volume when it is sliced in a particular way.

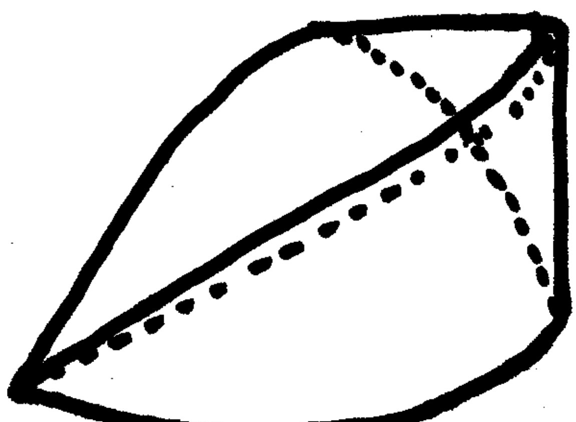
Example

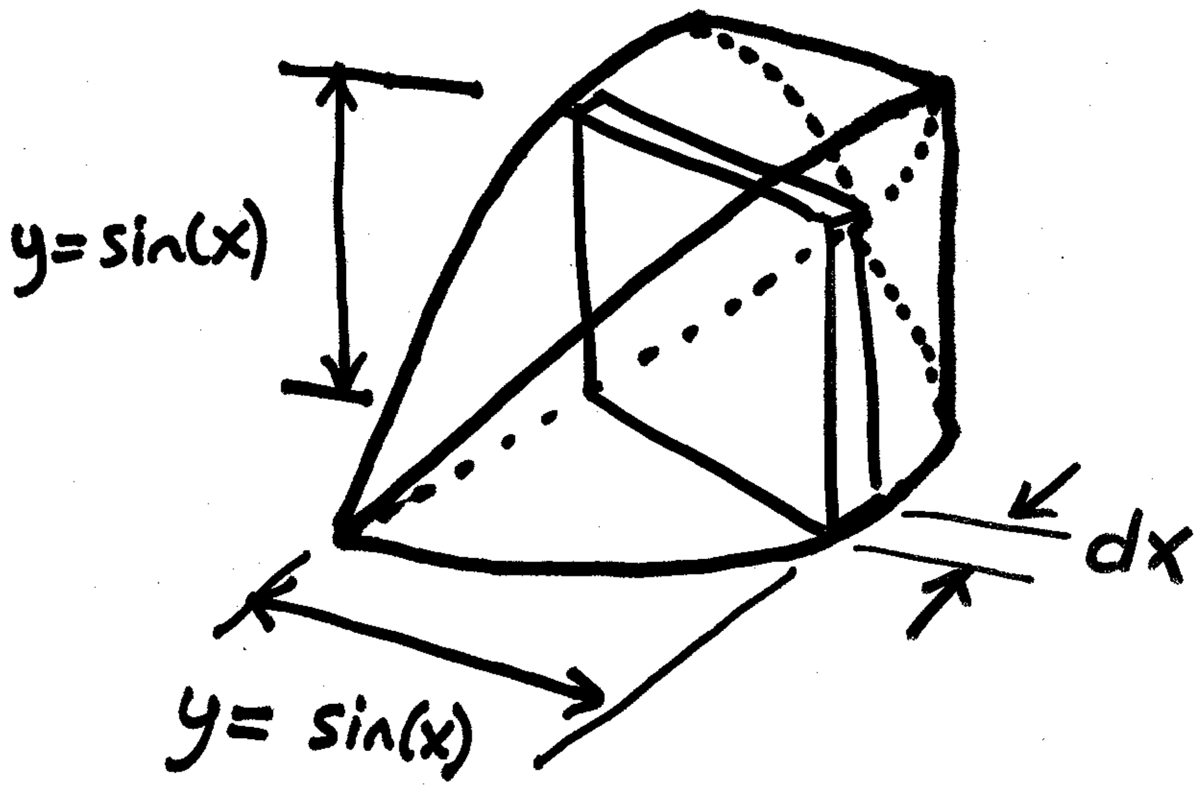
Base is the region bounded by: $y = 0$, $x = 0$, $y = \sin(x)$, $x = \pi$

Cross-sections perpendicular to the x-axis are squares.

Solution







$$\text{Volume of Slice} = y^2 \cdot dx$$

$$= \sin^2(x) \cdot dx$$

$$\text{Total Volume} = \int_0^{\pi} \sin^2(x)\, dx = \frac{\pi}{2}$$