Outline

1. Area between curves.
2. Volume by disks.
3. Volume by shells.

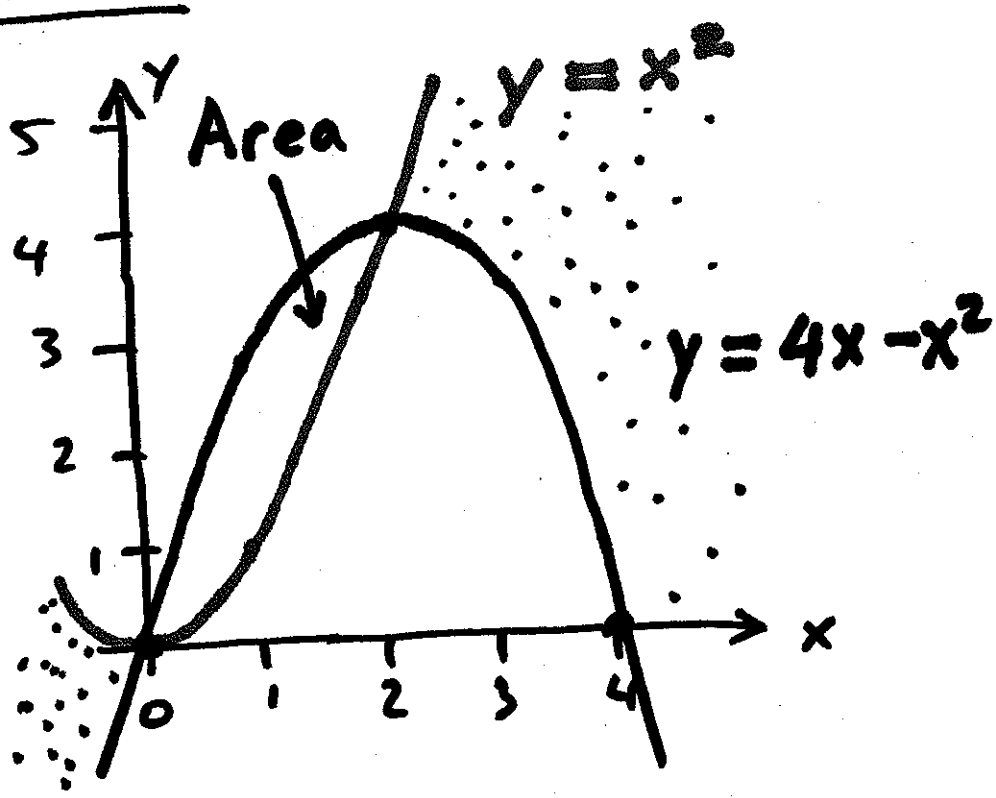
1. Areas Between Curves

Example

Find the area bounded by the curves:

$y = x^2$ and $y = 4x - x^2$.

Solution



To find the limits of integration:

$$x^2 = 4x - x^2$$

$$2x^2 = 4x$$

$$x^2 = 2x$$

One solution : $\boxed{x = 0}$ ← Limits of integration

If $x \neq 0$ $\quad x^2 = 2x$

$\boxed{x = 2}$ ← Limits of integration

$$\text{Area} = \int_0^2 4x - x^2 - x^2 \, dx$$

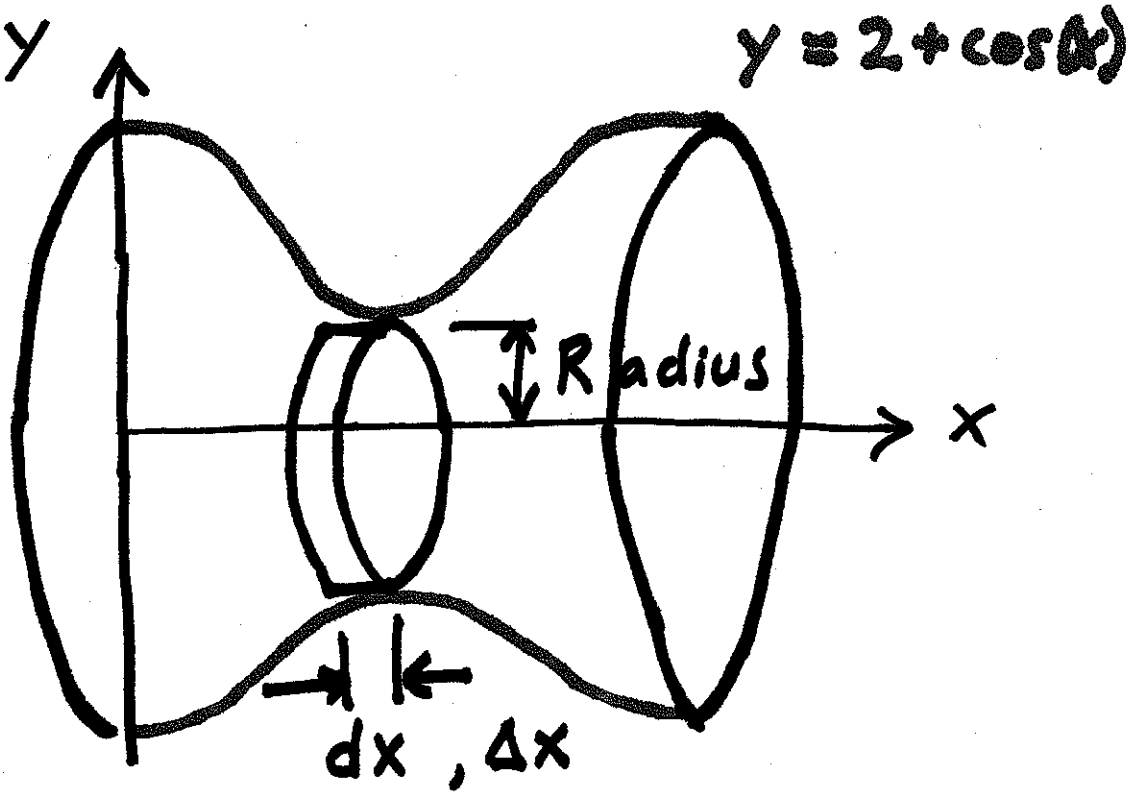
$$= \left[\frac{4}{2}x^2 - \frac{2}{3}x^3 \right]_0^2$$

$$= 8/3.$$

2. Volume of Revolution –

Disks

Idea:



Volume of slice = $\pi \cdot (\text{Radius})^2 \cdot dx$

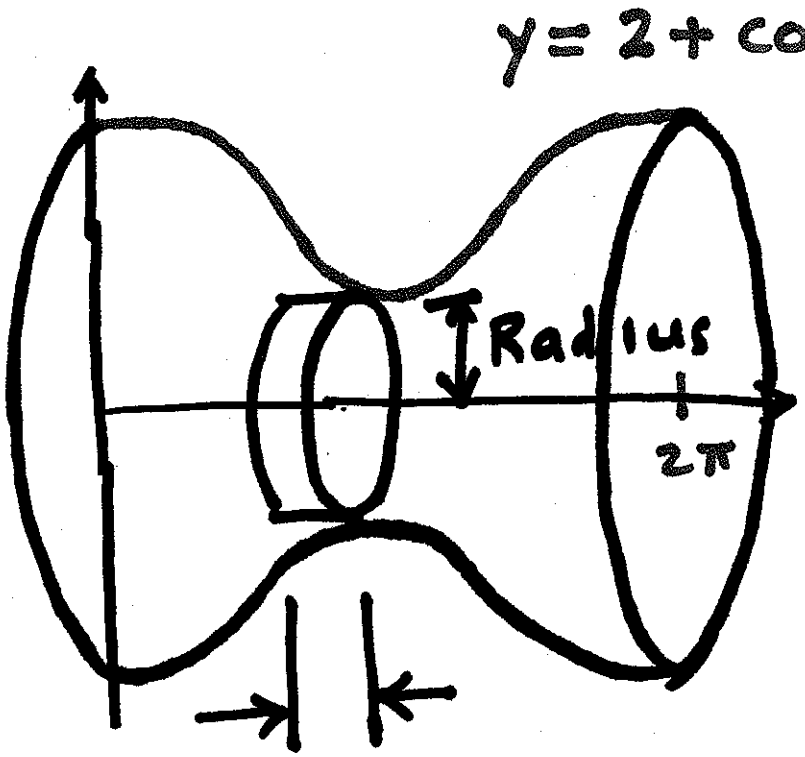
Total volume = $\int_a^b \pi \cdot [f(x)]^2 \cdot dx$

radius

wafer thin thickness

2. Volumes of Revolution

Idea:



Radius = $f(x)$

Area starts at $x = a$

Area ends at $x = b$

$$\text{Volume} = \int_a^b \underbrace{\pi \cdot f(x)^2 \cdot dx}_{\text{Formula for a wafer thin slice of solid.}}$$

Radius

Handout 6: Estimating the Psychoactive Dose of a Natural Hallucinogen

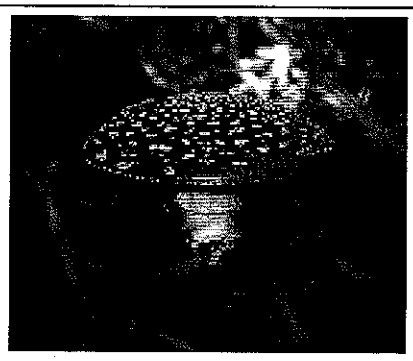


Figure 1: A mature specimen of *Amanita muscaria*.

The genus *Amanita* is a genus of mushrooms. Some species (such as *Amanita phalloides* and *Amanita virosa* which are both known by sinister common names such as "Death Cap" and "Destroying Angel") are highly toxic if eaten. These mushrooms are among the most toxic known, and as little as one half of a mushroom can kill a healthy adult within hours. The toxic compound found in these mushrooms is called "Amanitin" and while lethal to humans, many other mammals (such as squirrels) are completely immune to its effects[1]. Other species from the genus *Amanita* contain chemicals that have an intoxicating and hallucinogenic effect on humans. (Perhaps the most notorious is *Amanita muscaria*, commonly known as the "Fly agaric," for its ability to attract and kill flies – see Figure 1[2].) The chemical found in *Amanita muscaria* that has been most strongly linked to these effects is muscimol that closely resembles the neurotransmitter THIP (see Figure 2[3]).

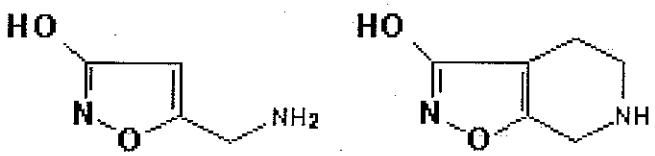


Figure 2: Molecular structure of muscimol (left) the psycoactive chemical found in *Amanita muscaria* and the neurotransmitter THIP (right).

A mature specimen of *Amanita muscaria* has a cap that is about 5cm high with a diameter of about 16 cm. The stalk is averages 11cm long with a radius of about 2cm. If a mature specimen of *Amanita muscaria* were cut right up the middle, and then cut right up the middle again, the "cross-section" revealed would look something like Figure 3 (below).

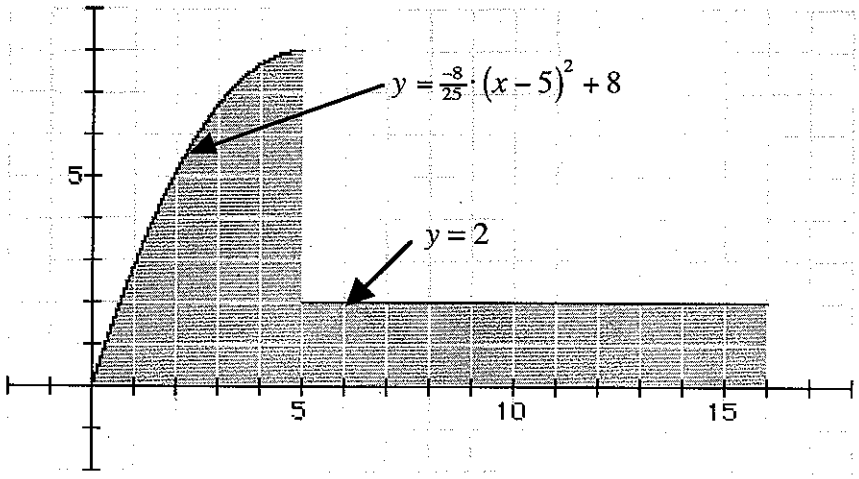


Figure 3: Mathematical model for a mature specimen of *Amanita muscaria*.

[1] Source: http://museum.gov.ns.ca/poison/angel.htm
[2] Image source: http://www.du.edu/~kinnamon/3640/neurotransmitters/
[3] Image source: http://staff-www.uni-marburg.de/

In this handout, you will ultimately work out how many *Amanita muscaria* mushrooms a person would have to consume in order to experience a state of altered consciousness. Naturally, the precise level will vary from individual to individual depending on a range of factors such as body mass, tolerance for muscimol, etc. It is important to point out that although no fatalities[4] have ever been directly connected to the consumption of *Amanita muscaria*, consumption of this mushroom may cause extreme physical discomfort (such as cramps and diarrhea) and has caused people to lapse into a coma when taken in very large doses[5]. Many of the people who have ingested *Amanita muscaria* have reported nausea and vomiting. Also important to note is the fact[6] that approximately 90% of the 10,000 mushroom poisonings reported in North America every year are the result of consumption of misidentified members of the genus *Amanita*.

(a) The first phase of this calculation will be to determine the volume of a mature specimen of *Amanita muscaria*. You can imagine the volume of the mushroom being formed by taking the shaded area from Figure 3 and revolving this around the x-axis. Draw a sketch to show the three-dimensional shape that will be formed when the area from Figure 3 is revolved around the x-axis.

(b) When calculating the volume of a solid of revolution, it is customary to break the complicated shape up into pieces that are much simpler. Draw some of these simpler shapes on the 3-D sketch of the solid of revolution that you made in Part (a).

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[4] See: D.R. Benjamin. (1992) "Mushroom poisoning in infants and children: The *Amanita pantherina/muscaria* group." *Journal of Toxicology and Clinical Toxicology*, **30**(1): 13-22.

[5] Source: C. Kuhn, S. Swartzwelder and W. Wilson. *Buzzed: The Straight Facts About The Most Used And Abused Drugs From Alcohol To Ecstasy*. New York: W.W. Norton and Company, 1998.

[6] Source: T.L. Litovitz, W. Klein-Schwartz, E.M. Caravati, J. Youniss, D. Crouch and S. Lee. (1999) "1999 Annual Report of the American Association of Poison Control Centers Toxic Exposure Surveillance System." *American Journal of Emergency Medicine*, **17**(5): 435-487.

(c) The outline of the bright red "cap" portion of the mushroom is represented by the equation:

$$y = \frac{-8}{25} \cdot (x-5)^2 + 8$$

which is valid between $x = 0$ and $x = 5$. Find a formula for the volume of the simple shapes that you sliced the "cap" of the mushroom into in Part (b).

(d) Create an integral that will give the volume of the bright red "cap" of the mushroom.

(e) Evaluate the integral from Part (d). The numerical result that you get will be the volume of the cap of the mushroom in units of cubic centimeters.

(f) Calculate the volume of the stalk of the mushroom in units of cubic centimeters.

(g) According to studies[7] carried out in the 1960's, a person needs to consume approximately 10-15 mg of muscimol before any intoxicating or hallucinogenic effects are typically experienced. The concentration of muscimol in *Amanita muscaria* fluctuates during the year and particularly during the mushroom's reproductive stages[8]. The "accepted wisdom" among *Amanita muscaria* enthusiasts is that the mushroom is best harvested during the first weeks of August[9]. At this time, the concentration of muscimol in *Amanita muscaria* is approximately[10] 0.008 mg per cubic centimeter. About how many mushrooms would you expect a person to have to consume in order to experience any intoxication or hallucinogenic effects?

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[7] Source: K.Bowden, A.C. Drysdale and G.A. Mogey. (1965) "Constituents of *Amanita muscaria*." *Nature*: 206: 1359-1360. and P.G. Waser. "The pharmacology of *Amanita muscaria*." In D.H. Efron, B. Homstead and N.S. Kline (eds.) *Ethnopharmalogic Search for Psychoactive Drugs*. Washington, DC: U.S. Government Printing Office, 1967.

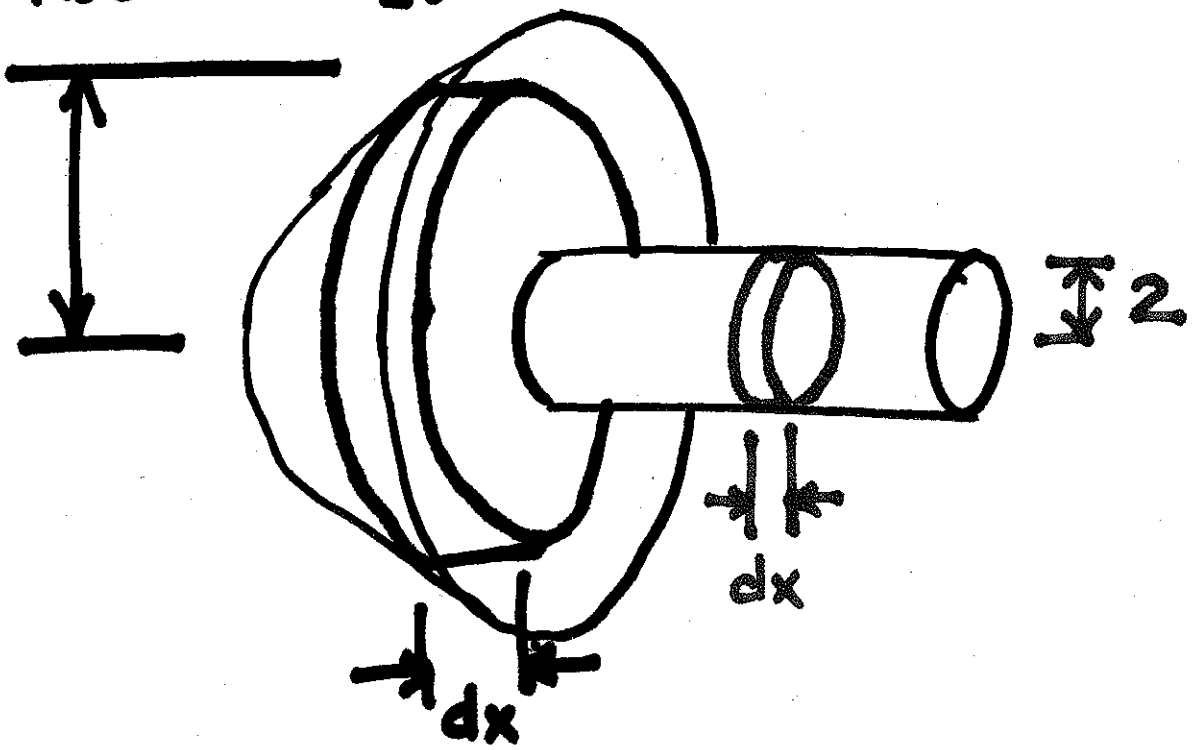
[8] Source: K. Tsunoda, N. Inoue, Y. Aoyagi and T. Sugahara. (1993) "Change in ibotenic acid and muscimol in the fruit body of *Amanita muscaria* during the reproductive stage." *Journal of the Food Hygiene Society of Japan*, 34(1): 18-24.

[9] Source: F. Festi and A. Bianchi. (1991) "*Amanita muscaria*: Myco-pharmacological outline and personal experiences." *Psychedelic Monographs and Essays*, 5:209-250.

[10] Source: P. Catalfomo and C.H. Eugster. (1970) "*Amanita muscaria*: Present understanding of its chemistry." *United Nations Office for Drug Control and Crime Prevention Bulletin on Narcotics*, 4: 33-41.

Example

Radius = $\frac{-8}{25}\cdot(x-5)^2 + 8$



Total volume of stalk $= \int_5^{16} \pi \cdot 2^2 \cdot dx$

Total volume of cap $= \int_0^5 \pi \cdot \left[\frac{-8}{25}(x-5)^2+8\right]^2 \cdot dx$

Cap volume = 536.17 cm^3

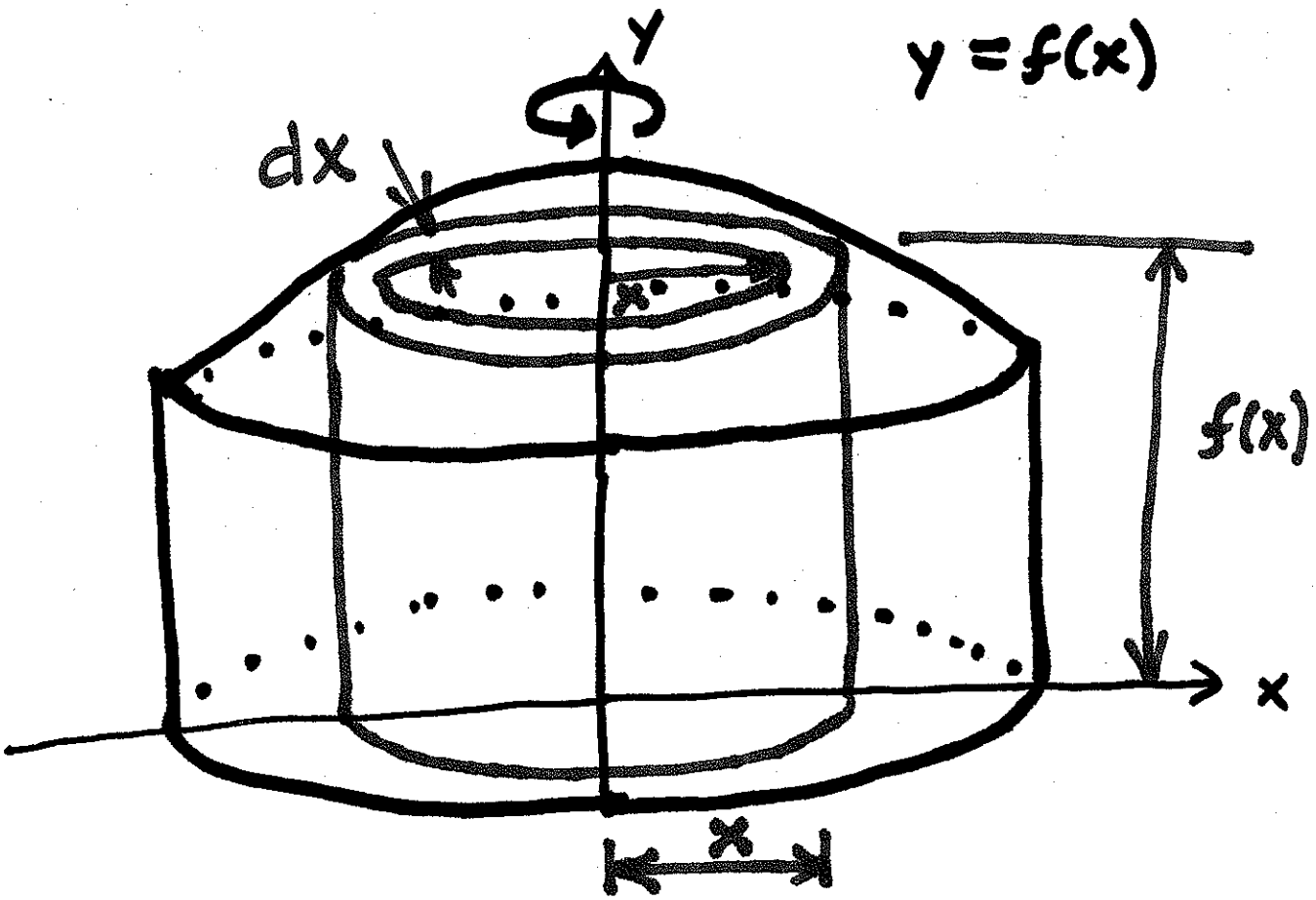
Stalk volume = 138.23 cm^3

Muscimol in mushroom =

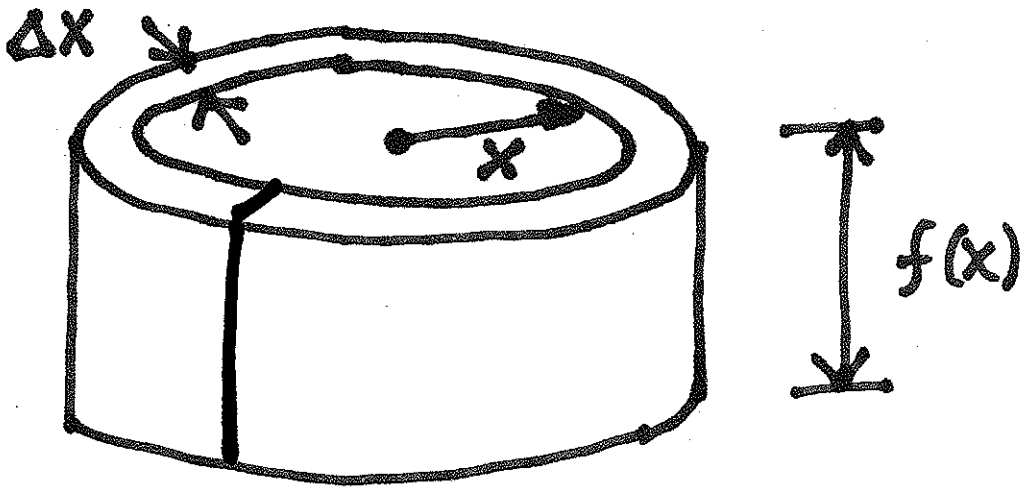
$0.008(536.17 + 138.23) = 5.395$ mg

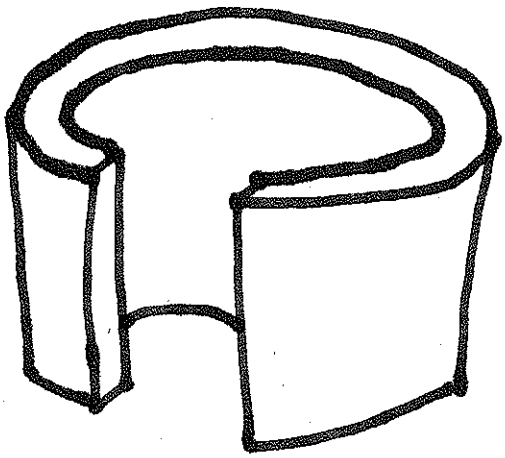
3. Volume of Revolution – Shells

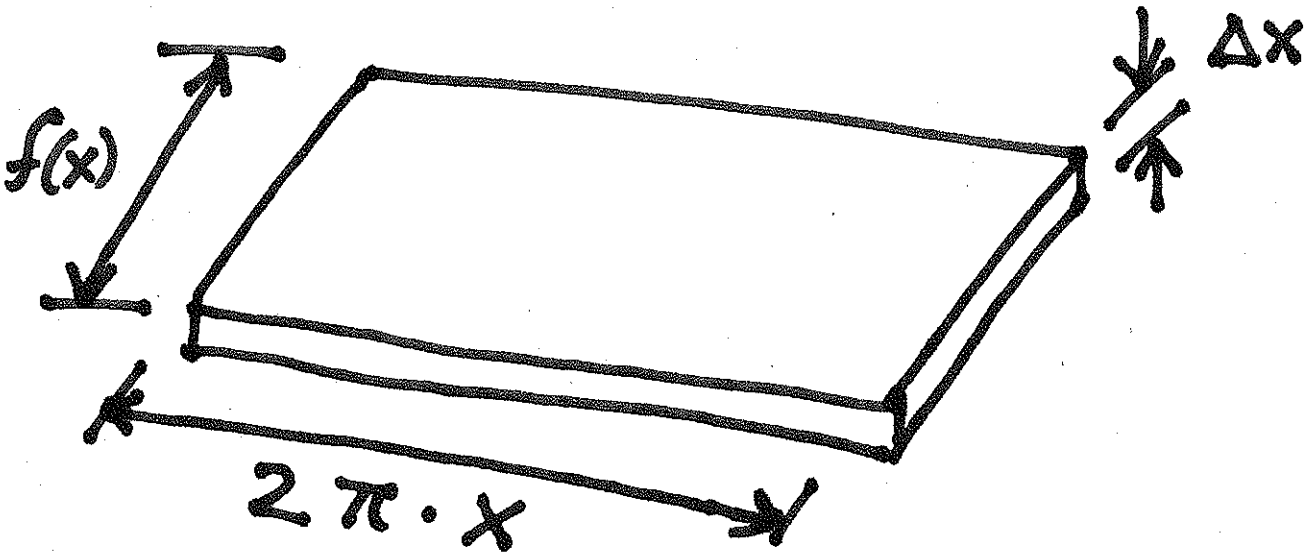
Idea: Break up irregular volume into regular shapes & create integral.



$$\text{Volume} = 2\pi x \cdot f(x) \cdot dx$$







Volume = $2\pi x \cdot f(x) \cdot \Delta x$