Solutions to Homework #9

Problems from Pages 436-437 (Section 8.3)

10. We begin by making a guess concerning the convergence or divergence of the series:

$$\sum_{n=2}^{\infty}\frac{\sqrt{n}}{n-1}.$$

My guess is that this series diverges. I am basing this guess on the observation that for large values of n,

$$\frac{\sqrt{n}}{n-1}\approx\frac{\sqrt{n}}{n}=\frac{1}{\sqrt{n}}.$$

Now, $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}}$ is a p-series with $p = 0.5 < 1$ so it diverges. I guess that $\sum_{n=2}^{\infty}\frac{\sqrt{n}}{n-1}$ does something similar (i.e. diverges also).

To use the Comparison test we will set $b_n=\frac{\sqrt{n}}{n-1}$ and build a series $\sum_{n=1}^{\infty}a_n$ that satisfies the following two conditions:

Condition I: $a_n < b_n$

Condition II: $\sum_{n=1}^{\infty}a_n$ diverges.

To build a_n we will start with the denominator of b_n and the observation that when $n > 1$:

$$n-1<n$$

so that,

$$\frac{1}{n}<\frac{1}{n-1}.$$

Multiplying both sides of this inequality by the positive $\sqrt{n}$ does not change the direction of the inequality sign and gives that:

$$\frac{1}{\sqrt{n}}<\frac{\sqrt{n}}{n}<\frac{\sqrt{n}}{n-1}.$$

Setting $a_n = \frac{1}{\sqrt{n}}$ ensures that Condition I is satisfied. To demonstrate that Condition II is satisfied, note that $\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ which is a p-series with $p = 0.5 < 1$ ensuring that it diverges.

As Conditions I and II are met by $a_n = \frac{1}{\sqrt{n}}$, we can conclude that the Comparison Test shows that the infinite series $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$ diverges.

14. We begin by making a guess concerning whether the infinite series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ converges or diverges. I guess that the infinite series diverges. I base this guess on the fact that for large values of n,

$$\frac{n^2}{n^3+1} \approx \frac{n^2}{n^3} = \frac{1}{n}.$$

The infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ is a p-series with p =1 so it diverges. Because of the similarity between the terms of this series and the terms of the more complicated series we are investigating, I guess that $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ also diverges.

To demonstrate that the infinite series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ actually does diverge, we will use the Comparison Test. To do this we will set $b_n = \frac{n^2}{n^3+1}$ and build a series $\sum_{n=1}^{\infty} a_n$ that satisfies the following two conditions:

Condition I: $a_n < b_n$

Condition II: $\sum_{n=1}^{\infty} a_n$ diverges.

To build a_n we will start with the denominator of b_n and the observation that when $n > 1$:

$$n^3 + 1 < n^3 + n^3 = 2 \cdot n^3.$$

Taking reciprocals gives:

$$\frac{1}{2\cdot n^3} < \frac{1}{n^3+1}.$$

Multiplying both sides of this inequality by the positive n^2 preserves the direction of the less than sign giving,

$$\frac{1}{2}\cdot\frac{1}{n} < \frac{n^2}{2\cdot n^3} < \frac{n^2}{n^3+1}.$$

Setting $a_n = \frac{1}{2}\cdot\frac{1}{n}$ gives that $a_n < b_n$ satisfying Condition I. To demonstrate that Condition II is also satisfied, note that:

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{2}\cdot\frac{1}{n} = \sum_{n=1}^{\infty} \frac{\frac{1}{2}}{n},$$

which is a p-series with $p = 1$. As such, $\sum_{n=1}^{\infty} a_n$ diverges so by the Comparison Test we may conclude that $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ also diverges.

16. We begin with a guess as to whether the infinite series $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$ converges or diverges. I guess that this series converges. I base this on the observation that when n is large,

$$\frac{n^2-1}{3n^4+1} \approx \frac{n^2}{3\cdot n^4} = \frac{\frac{1}{3}}{n^2}.$$

The infinite series $\sum_{n=1}^{\infty} \frac{\frac{1}{3}}{n^2}$ is a p-series with $p = 2 > 1$ so it converges. Because of the similarity in behavior between the terms of this series and the more complicated one we are investigating here, I guess that the series $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$ also converges.

To demonstrate the convergence of the infinite series $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$ we will use the Comparison Test. To do this we will set $a_n = \frac{n^2-1}{3n^4+1}$ and build an infinite series $\sum_{n=1}^{\infty} b_n$ that satisfies both of the following conditions:

Condition I: $a_n < b_n$

Condition II: $\sum_{n=1}^{\infty} b_n$ converges.

To begin this process we will start with the denominator of a_n and the following inequality:

$$3n^4 < 3n^4 + 1.$$

Taking reciprocals reverses the direction of the inequality and gives:

$$\frac{1}{3n^4+1} < \frac{1}{3n^4}.$$

Now, if $n > 1$ then $0 < n^2 - 1 < n^2$ so that:

$$a_n = \frac{n^2-1}{3n^4+1} < \frac{n^2}{3n^4+1} < \frac{n^2}{3n^4} = \frac{\frac{1}{3}}{n^2}.$$

Setting $b_n = \frac{1}{3} \cdot \frac{1}{n^2}$ gives us the terms of a series that satisfy Condition I. Noting that:

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{\frac{1}{3}}{n^2}$$

shows that Condition II is also satisfied because this is a p-series with $p = 2 > 1$, and so convergent. As Conditions I and II have both been satisfied with the choice of $b_n = \frac{1}{3} \cdot \frac{1}{n^2}$, the Comparison Test gives that the infinite series $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$ converges.

20. We begin this problem with the customary guess concerning the convergence or divergence of the given infinite series, in this case, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$. I guess that this infinite series converges. The reason for this guess is the observation that for large values of n,

$$\frac{1}{\sqrt{n^3+1}} \approx \frac{1}{\sqrt{n^3}} = \frac{1}{n^{\frac{3}{2}}}.$$

The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ is a p-series with $p = 1.5 > 1$. This series converges, and so I guess that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ also converges because the terms in the two series are very similar when n is large.

To formalize the demonstration that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ converges, we will use the Comparison Test. To do this we will set $a_n = \frac{1}{\sqrt{n^3+1}}$ and build an infinite series $\sum_{n=1}^{\infty} b_n$ that satisfies both of the following conditions:

Condition I: $a_n < b_n$

Condition II: $\sum_{n=1}^{\infty} b_n$ converges.

To begin this process we will start with the contents of the square root from the denominator of a_n and the following inequality:

$$n^3 < n^3 + 1.$$

As the square root function is a monotonically increasing function, taking square roots of both sides does not change the direction of the inequality:

$$\sqrt{n^3} < \sqrt{n^3+1}.$$

Taking reciprocals does change the direction of the inequality, leaving us with:

$$\frac{1}{\sqrt{n^3+1}} < \frac{1}{\sqrt{n^3}} = \frac{1}{n^{\frac{3}{2}}}.$$

We will set $b_n = \frac{1}{\sqrt{n^3}}$. The inequality immediately above shows that $a_n < b_n$ so that Condition I is satisfied by this choice. Furthermore, the series $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ is a p-series with $p = 1.5 > 1$. This series converges, and so Condition II is satisfied by the choice of $b_n = \frac{1}{\sqrt{n^3}}$. With Conditions I and II both satisfied, the Comparison Test gives that the infinite series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ converges.

24. This last problem from Section 8.3 will also be approached by initially making a guess concerning the convergence or divergence of the infinite series, $\sum_{n=0}^{\infty} \frac{1+\sin(n)}{10^n}$. I guess that this series converges. The reasoning behind this guess is: the numerator is bounded above by 2 whereas the denominator grows rapidly as n increases.

To demonstrate convergence formally we will use the Comparison Test. To do this, set $a_n = \frac{1+\sin(n)}{10^n}$. We will build an infinite series $\sum_{n=0}^{\infty} b_n$ that satisfies both of the following conditions:

Condition I: $a_n < b_n$

Condition II: $\sum_{n=0}^{\infty} b_n$ converges.

To begin this process note that for any value of n, $0 \leq 1+\sin(n) \leq 1$ so that:

$$1+\sin(n) < 2.$$

For any value of n, 10^n is a positive number so dividing both sides of the inequality show immediately above by 10^n will not alter the direction of the inequality sign. Doing this gives:

$$\frac{1+\sin(n)}{10^n} < \frac{2}{10^n}.$$

Setting $b_n = 2 \cdot \left(\frac{1}{10}\right)^n$, the inequality immediately above shows that this choice satisfies Condition I's requirement that $a_n < b_n$. Furthermore, noting that $\sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty} 2 \cdot \left(\frac{1}{10}\right)^n$ is a geometric series with $r = 0.1$ (and hence convergent) gives that Condition II also holds. As both Condition I and Condition II hold, the Comparison Test gives that the infinite series $\sum_{n=0}^{\infty} \frac{1+\sin(n)}{10^n}$ converges.

Problems from Pages 446-447 (Section 8.4)

10. Convergence of the series

The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 5^n}$ is an alternating series. The factor of $(-1)^n$ produces the change in sign from term to term. To establish the *conditional* convergence of this series, we can use the Alternating Series Test for Conditional Convergence. This particular series also happens to be absolutely convergent so for the sake of variety, instead of using the alternating series test, we will use the Ratio test to demonstrate absolute convergence (and absolute convergence implies conditional convergence).

The Ratio Test

Set $b_n = \frac{(-1)^n}{n \cdot 5^n}$. Then the ratio that we need to set up and simplify is:

$$\frac{b_{n+1}}{b_n} = \frac{(-1)^{n+1}}{(n+1) \cdot 5^{n+1}} \cdot \frac{n \cdot 5^n}{(-1)^n} = \frac{-n}{5n+5}.$$

Taking the limit of $\left|\frac{b_{n+1}}{b_n}\right|$ as $n \to \infty$ gives a result of 0.2. As the limit is less than one, the Ratio test gives that the infinite series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 5^n}$ is absolutely convergent, and hence also conditionally convergent.

Number of terms to approximate the sum of the series

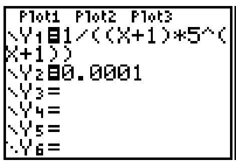
Let S_N represent the N^{th} partial sum of the infinite series $\sum_{n=1}^{\infty} (-1)^n \cdot a_n$ (that is, S_N is the sum of the first N terms of the series added together, $S_N = \sum_{n=1}^{N} (-1)^n \cdot a_n$). Let S represent the sum of the infinite series, i.e. $S = \sum_{n=1}^{\infty} (-1)^n \cdot a_n$.

The error involved in using S_N to approximate S is bounded above by a_{N+1}.

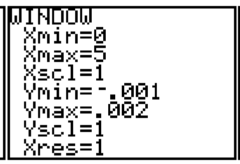
To determine the value of N that should be used to estimate $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 5^n}$ with $S_N = \sum_{n=1}^{N} \frac{(-1)^n}{n \cdot 5^n}$ incurring an error of less than 0.0001, we will solve the equation:

$$a_{N+1} = \frac{1}{(N+1) \cdot 5^{N+1}} < 0.0001.$$

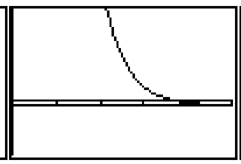
This is a very difficult equation to solve algebraically so we will find an approximate solution using a graphing calculator and then round the value of N that we get up to the nearest whole number. The steps involved in solving this problem using a graphing calculator are shown below.

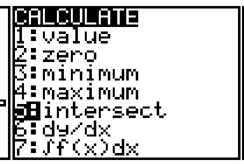


Enter the two sides of the inequality as two separate functions into the calculator.

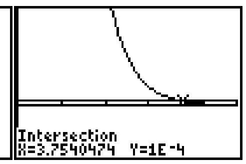


Set the size of the graphing window so that the intersection point of Y1 and Y2 will show clearly on the calculator screen.


Display the two graphs on the calculator screen. Make sure the intersection point is visible.



Use the calculator's ability to find an intersection point to locate the coordinates of the intersection point.



The x-coordinate of the intersection point is the value of N that we are looking for.

In this case, the solution of the equation is $x \approx 3.75$ so rounding this up gives that using $N = 4$ should give an approximation to $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 5^n}$ that has an error of less than 0.0001.

16. In order to approximate the sum of the series $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n \cdot n!}$ by a partial sum $S_N = \sum_{n=1}^{N} \frac{(-1)^n}{3^n \cdot n!}$ with an error of less than 0.000005 (which will safely guarantee four decimal places of accuracy), we need to determine the value of N that we should sum to.

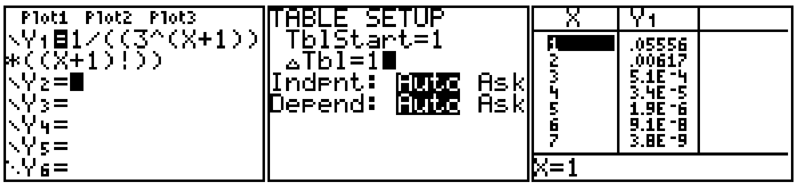
As noted earlier, the error involved in using S_N to approximate S is bounded above by a_{N+1}, i.e. the absolute value of the $(N+1)^{th}$ term of the series.

Determine N for the partial sum

Set $a_n = \dfrac{1}{3^n \cdot n!}$. To determine the value of N that we need, we must solve the equation:

$$a_{N+1} = \frac{1}{3^{N+1} \cdot (N+1)!} < 0.000005 = 5 \times 10^{-6}.$$

This is a very difficult problem to solve using algebra, so we will solve it by making a table on a graphing calculator. The steps involved in doing this on a TI-84 calculator are shown in the diagram below.



Enter the left hand side of the inequality as Y1 in the calculator.	Press [2nd][TBLSET] and set TblStart to 1 and ΔTbl to 1.	Press [2nd][TABLE] and look down the Y1 column for the first value that is less than 0.00001. The corresponding x-value is the value of *N*.

The calculator shows that we should use $N = 5$ to ensure the level of accuracy desired. You can either add up the first 5 terms of the series by hand or have the calculator do it. The result of this computation is given below.

$$S \approx S_5 = \sum_{n=1}^{5} \frac{(-1)^n}{3^n \cdot n!} = -0.2834.$$

22. To determine whether or not the infinite series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{n^2+1}$ converges *conditionally*, we can use the Alternating Series Test for Conditional convergence. This test is applicable because this infinite series has the factor of $(-1)^n$ which causes the terms to alternate sign.

Setting $a_n = \dfrac{n}{n^2+1}$, the two conditions that must be met for the Alternating Series Test are:

Condition I: $a_{n+1} < a_n$.

Condition II: The limit of a_n as $n \to \infty$ must be zero.

Demonstrating that Condition I holds:

We will begin the demonstration that Condition I holds with the rather unlikely but nevertheless valid (when $n > 1$) inequality:

$$n^3 + n^2 + n + 1 < n^3 + 2n^2 + 2n.$$

Factoring both sides of the above inequality gives:

$$(n+1)\cdot\left(n^2+1\right) < n\cdot\left((n+1)^2+1\right).$$

Dividing both sides by the positive quantity $\left(n^2+1\right)\cdot\left((n+1)^2+1\right)$ does not affect the direction of the inequality sign and gives us the inequality we require to establish Condition I:

$$a_{n+1} = \frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1} = a_n.$$

Demonstrating the Condition II holds:

The limit of a_n as $n \to \infty$ is equal to zero because the largest power of n in the denominator of a_n is greater than the largest power of n in the numerator.

Since Conditions I and II have both been met, the Alternating Series Test gives that the infinite series $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{n}{n^2+1}$ converges conditionally.

Absolute Convergence

The infinite series $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{n}{n^2+1}$ does not converge absolutely. The question of the absolute convergence of this series is equivalent to the question of the convergence of the infinite series: $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$. This can be investigated using the Integral Test.

Performing the Integral Test

We will set $f(x) = \frac{x}{x^2+1}$. In order to use the Integral Test we must first verify that $f(x)$ obeys the two necessary conditions:

(i) $f(x) > 0$ (ii) $f'(x) < 0$

for $x > 1$.

Condition (i) holds because the numerator (x) and denominator ($1 + x^2$) of $f(x)$ are both positive when $x > 1$.

Condition (ii) hold because when we use the Quotient Rule to differentiate $f(x)$ we get:

$$f'(x) = \frac{x^2 + 1 - 2x \cdot x}{\left(1 + x^2\right)^2} = \frac{1 - x^2}{\left(1 + x^2\right)^2}.$$

When $x > 1$, the quantity $1 - x^2$ is negative so the numerator of $f'(x)$ is negative while the denominator is positive. This makes the sign of $f'(x)$ negative overall.

To carry out the integral test we must calculate the improper integral $\int_1^\infty \frac{x}{1 + x^2}\,dx$, which we will do with the help of the u-substitution $u = 1 + x^2$.

$$\int_1^\infty \frac{x}{1 + x^2}\,dx = \lim_{a \to \infty} \int_1^a \frac{x}{1 + x^2}\,dx = \lim_{a \to \infty} \left[\tfrac{1}{2} \cdot \ln\left(1 + x^2\right)\right]_1^a = \lim_{a \to \infty} \tfrac{1}{2} \cdot \ln\left(1 + a^2\right) - \tfrac{1}{2} \cdot \ln(2) = +\infty$$

The improper integral diverges so the Integral Test says that the infinite series $\sum_{n=1}^\infty \frac{n}{n^2 + 1}$ diverges also. This means that the series $\sum_{n=1}^\infty (-1)^n \cdot \frac{n}{n^2 + 1}$ does not converge absolutely.

32. To determine whether or not the infinite series $\sum_{n=2}^\infty \frac{(-1)^n}{n \cdot \ln(n)}$ converges *conditionally*, we can use the Alternating Series Test for Conditional convergence. This test is applicable because this infinite series has the factor of $(-1)^n$ which causes the terms to alternate sign.

Setting $a_n = \frac{1}{n \cdot \ln(n)}$, the two conditions that must be met for the Alternating Series Test are:

Condition I: $a_{n+1} < a_n$.

Condition II: The limit of a_n as $n \to \infty$ must be zero.

Demonstrating that Condition I holds:

Note that $f(x) = \ln(x)$ is an increasing function, and that $n + 1 > n$. Therefore:

$$\ln(n+1) > \ln(n),$$

so that:

$$\frac{1}{\ln(n+1)} < \frac{1}{\ln(n)}.$$

Now, since $n + 1 > n$, we can conclude that when $n > 0$, $\frac{1}{n+1} < \frac{1}{n}$. Combining this with the inequality given above gives:

$$a_{n+1} = \frac{1}{(n+1)\cdot\ln(n+1)} < \frac{1}{n\cdot\ln(n+1)} < \frac{1}{n\cdot\ln(n)} = a_n,$$

so that Condition I is satisfied.

Demonstrating the Condition II holds:

When $n > 2$, note that $\ln(n) > 1$ so that we have the inequality:

$$0 < \frac{1}{n\cdot\ln(n)} < \frac{1}{n}.$$

The limits of both zero and $\frac{1}{n}$ are equal to zero as $n \to \infty$ so the Squeezing Lemma gives that the limit of a_n (as $n \to \infty$) is equal to zero. This shows that Condition II holds. By the Alternating Series Test, the infinite series $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\cdot\ln(n)}$ converges conditionally.

Absolute Convergence

There is a possibility that the infinite series $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\cdot\ln(n)}$ could converge absolutely. We will investigate this question by testing the infinite series $\sum_{n=2}^{\infty}\frac{1}{n\cdot\ln(n)}$ to see it is converges.

Noting that the technique of u-substitution ($u = \ln(x)$) gives that:

$$\int\frac{1}{x\cdot\ln(x)}\cdot dx = \ln(|\ln(|x|)|) + C$$

suggests that the Integral test might be a good test to investigate the convergence or divergence of the infinite series $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$.

Performing the Integral Test

Set $f(x) = \frac{1}{x \cdot \ln(x)}$. When $x > 1$, $\ln(x) > 1$ so that $f(x) > 0$. To verify that $f(x)$ is decreasing, note that:

$$f'(x) = \frac{-(\ln(x)+1)}{(x \cdot \ln(x))^2}.$$

So long as $\ln(x) > -1$, the derivative is negative and $f(x)$ is decreasing. Both conditions needed for the Integral Test are met by $f(x)$. All that remains is to compute the improper integral of $f(x)$ to determine whether it converges or diverges.

$$\int_2^{\infty} \frac{1}{x \cdot \ln(x)} \cdot dx = \underset{a \to \infty}{\mathrm{Lim}} \int_2^{a} \frac{1}{x \cdot \ln(x)} \cdot dx = \underset{a \to \infty}{\mathrm{Lim}} \left[\ln(|\ln(|x|)|) \right]_2^a = +\infty.$$

The improper integral diverges, so the infinite series $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$ also diverges. This means that the infinite alternating series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \cdot \ln(n)}$ does not converge absolutely.

40. To determine the positive integers k for which the infinite series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ converges, we will pursue the following strategy:

(a) We will carry out the Ratio test to determine what the limit of $\left|\frac{a_{n+1}}{a_n}\right|$ is as $n \to \infty$.

(b) We anticipate that the result of Part (a) will be a formula involving k.

(c) We will set analyze the result from Part (a) to determine when it is less than one (so that convergence is guaranteed by the Ratio test) and note the values of k for which this happens.

Carrying out Part (a) of the plan:

In this problem, $a_n = \dfrac{(n!)^2}{(kn)!}$ so that:

$$\frac{a_{n+1}}{a_n} = \frac{((n+1)!)^2}{(kn+k)!} \cdot \frac{(kn)!}{(n!)^2} = \frac{(n+1)^2}{(kn+k)\cdot(kn+k-1)\cdot\ldots\cdot(kn+1)}.$$

Carrying out Part (b) of the plan:

Now, the limit of this ratio will depend on the value of *k* as it is the value of *k* that controls the number of individual linear factors that appear in the denominator.

Carrying out Part (c) of the plan:

If $k = 1$ then $\frac{a_{n+1}}{a_n} = n+1$ so that the limit of $\left|\frac{a_{n+1}}{a_n}\right|$ as $n \to \infty$ is $+\infty$. Since this limit is greater than one, the Ratio test gives that the infinite series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ diverges when $k = 1$.

If $k = 2$ then $\frac{a_{n+1}}{a_n} = \frac{n+1}{2\cdot(2n+1)}$ so that the limit of $\left|\frac{a_{n+1}}{a_n}\right|$ as $n \to \infty$ is 0.25. Since this limit is less than one, the Ratio test gives that the infinite series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ converges when $k = 2$.

If $k > 2$, then,

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)}{k\cdot(kn+k-1)\cdot\ldots\cdot(kn+1)}.$$

Since the power of *n* in the denominator will be higher than the power of *n* in the numerator when $k > 2$, the limit of $\left|\frac{a_{n+1}}{a_n}\right|$ as $n \to \infty$ will be zero. Since this limit is less than one, the Ratio test gives that the infinite series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ converges when $k > 2$.

Final Answer: The infinite series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$ converges when $k > 2$.