Homework Assignment #7

Remember: Your solutions are due at the start of recitation on 10/28/08.

1. In each case, verify that the given function is a solution of the given differential equation.

(a) $t \cdot y' - y = t^2$ $\qquad y(t) = 3t + t^2$

(b) $y' - 2t \cdot y = 1$ $\qquad y(t) = e^{t^2} \cdot \int_0^t e^{-s^2}\, ds + e^{t^2}$

2. The most general initial value problem involving a first order linear differential equation (with constant coefficients) is:

$$\frac{dy}{dt} = ry + k \qquad y(0) = y_0$$

where r and k are constants. Find the solution of this initial value problem using the technique of Separation of Variables.

3. Find the solution of the following initial value problem. In your solution, clearly state the solution method that you have employed.

$$y' - y = 2te^{2t} \qquad y(0) = 1.$$

4. Find the solution of the following initial value problem. In your solution, clearly state the solution method that you have employed.

$$y' + \tfrac{2}{t} y = \tfrac{\cos(t)}{t^2} \qquad y(\pi) = 0,\ t > 0$$

5. Find the solution of the following initial value problem. In your solution, clearly state the solution method that you have employed.

$$t^3 \cdot y' + 4t^2 \cdot y = e^{-t} \qquad y(-1) = 0$$

6. Find the solution, $y(x)$, of the following initial value problem.

$$y'' + y' - 2y = 0 \qquad y(0) = 1 \qquad y'(0) = 1.$$

7. Find the solution, $y(x)$, of the following initial value problem.

$$2y'' + y' - 4y = 0 \qquad y(0) = 0 \qquad y'(0) = 1.$$

8. Find particular solutions of each the following differential equations:

(a) $y'' - 3y' - 4y = 3e^{2t}$.

(b) $y'' - 3y' - 4y = 2 \cdot \sin(t)$.

9. Find the solution, $y(x)$, of the following initial value problem.

$$y'' + 4y = t^2 + 3e^t \qquad y(0) = 0 \qquad y'(0) = 2.$$

10. Suppose that a room containing 1200 cubic feet of air is initially free of carbon monoxide. Beginning at time $t = 0$ cigarette smoke (containing 4% carbon monoxide) is introduced to the room at a rate of 0.1 cubic feet per minute. The well-circulated mixture is allowed to leave the room at the same rate.

(a) Find and expression for the concentration $x(t)$ of carbon monoxide in the room at any time $t > 0$.

(b) Extended exposure to carbon monoxide in concentrations as low as 0.00012 is harmful to the human body. Find the amount of time that elapses before this concentration is reached in the room.