Handout 7: Creating a Differential Equation

In the first part of today's recitation, you will participate in an experiment that simulates the spread of an infectious, incurable and non-lethal disease. An example of an actual disease that has these attributes is herpes.

The diagram on the next page shows how the activity will be performed. During this activity one of your recitation instructor will coordinate the movement and counting of the people in the circle and record the data as it is collected.

1. The class is formed into a large, oblong circle and one person starts with a bag of "disease pathogens."

2. When given a signal, everyone shuffles around three spaces.

3. If, at the end of this move, a person who is already infected with the disease is facing a person who does not have the "disease" then the "sick" person infects the healthy person by sharing some of his or her disease pathogens.

4. When all of the "infecting" has been performed, the number of people who have the disease at that time are counted and this number recorded.

5. The people at the rounded ends of the circle cannot infect or be infected.

6. The activity is repeated until everyone has the disease.

Time	Number of people infected	Time	Number of people infected
0	0	7	
1	0	8	
2	1	9	
3		10	
4		11	
5		12	
6		13	

Table 1

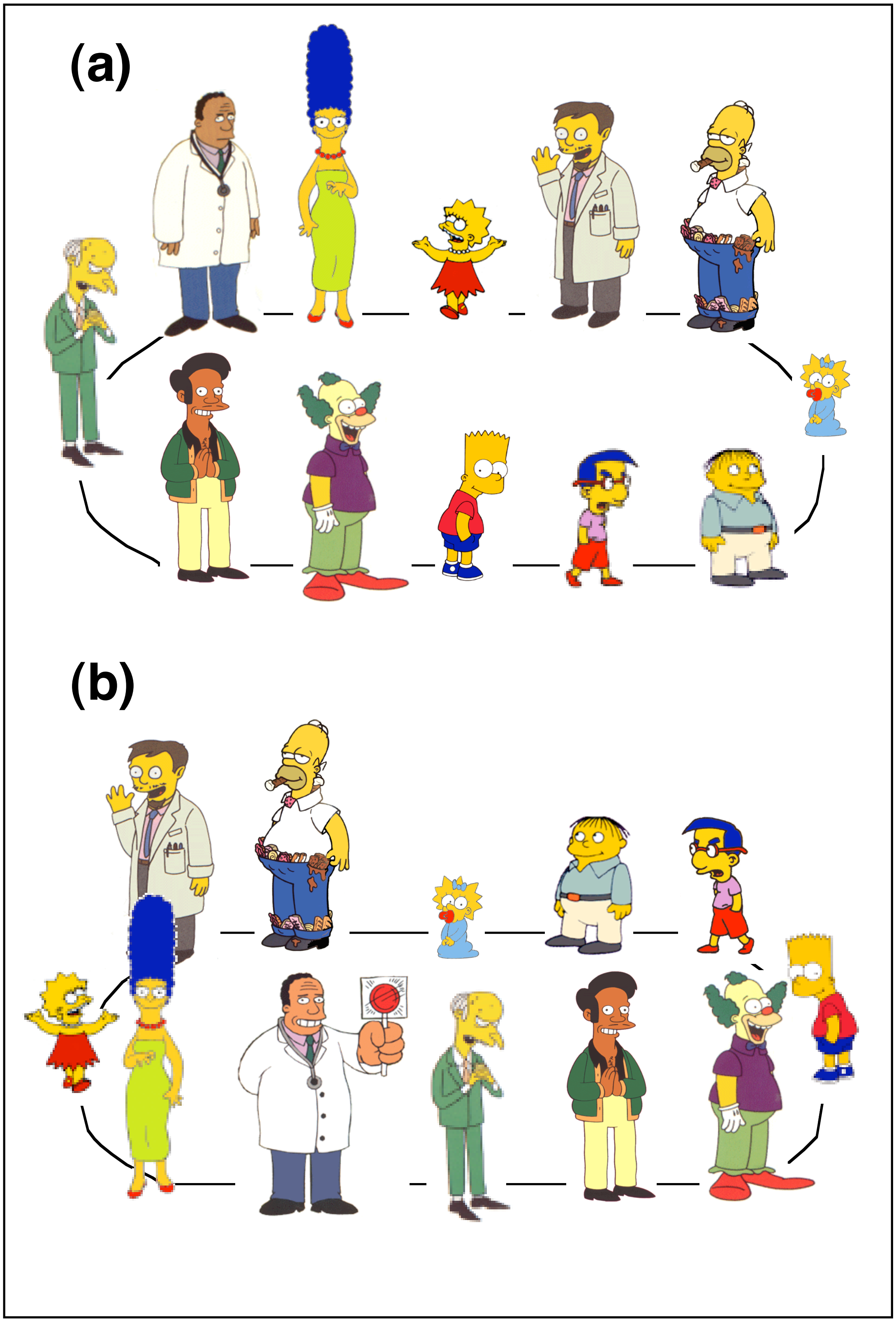
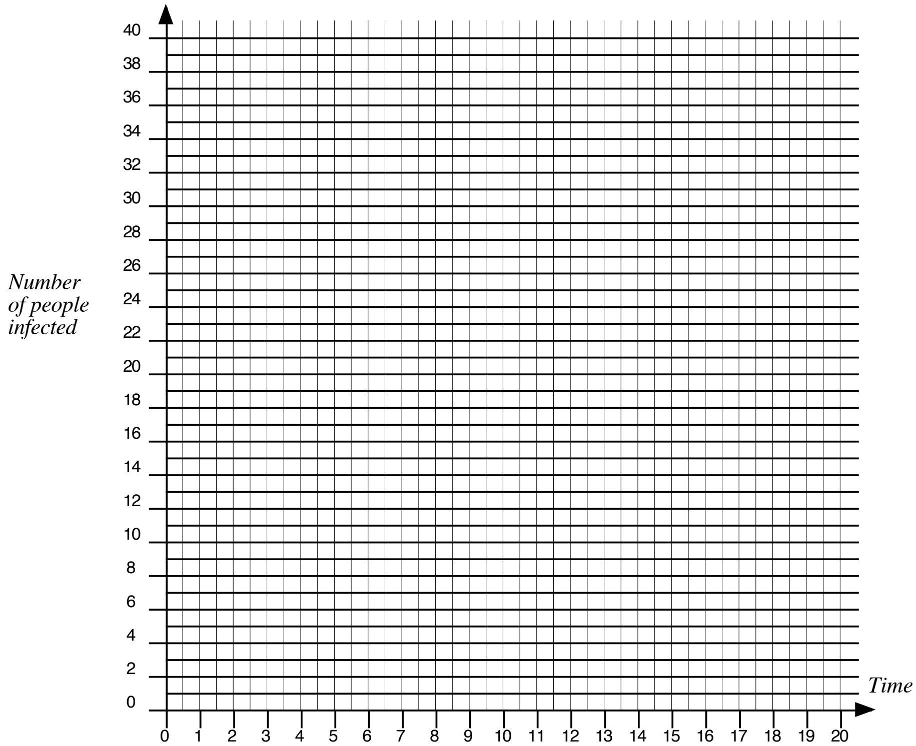


Figure 8: (a) Everybody in the class forms a large, oblong circle. To begin with, only one person in the class is infected with the disease (Homer). (b) When told to do so, everyone moves three spaces around the circle. When the movement is finished, diseased people (Homer) give the disease to the person opposite (Dr. Hibbert) by sharing their supply of disease pathogen.

Part 1: Representing and Understanding the Data

(a) Use the axes provided below to plot a graph showing the number of people infected with the disease as a function of time.



(b) The graph of your data will show two distinct "zones" of concavity (see below). At the beginning your graph will be concave up, but towards the end your graph will be concave down. In terms of what was going on during the activity, explain why these two different concavities occur.

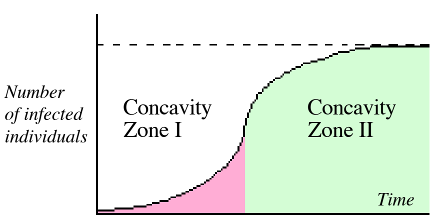


Figure 9: The two "zones" of concavity in your data.

Part 2: Creating a Differential Equation

You may have wondered where some of the differential equations that we study in class actually come from. In this section of the recitation you will use the data that you analyzed in Part 1 to create a differential equation that describes the spread of the disease.

The method that you will use is a common method of creating a differential equation from data. This method utilizes the regression capabilities of your calculator to find equations. If you are not sure how to do this, your recitation instructor has a handout explaining how to enter data and create regression equations that will work on a TI-84 calculator. The steps involved in creating the differential equation are listed below.

1. Calculate approximate values for the derivative using the data that you have collected and the difference quotient.

2. Plot a graph using the derivative as the *y*-value and the number of people infected as the *x*-value.

3. Find an equation to represent the relationship shown in this graph.

4. Re-write the equation that you have found as a differential equation.

From this point on, we will use:

- T to represent the time.

- $P(T)$ to represent the number of people who have been infected with the disease.

When the length of the time interval, ΔT, is small the derivative of the function $P(T)$ is approximately equal to the difference quotient.

$$P'(T) \approx \frac{P(T+\Delta T) - P(T)}{\Delta T}.$$

In particular, when the length of the time interval is equal to one (i.e. $\Delta T = 1$), the value of the derivative is roughly approximated by the difference quotient:

$$P'(T) \approx \frac{P(T+1) - P(T)}{1}.$$

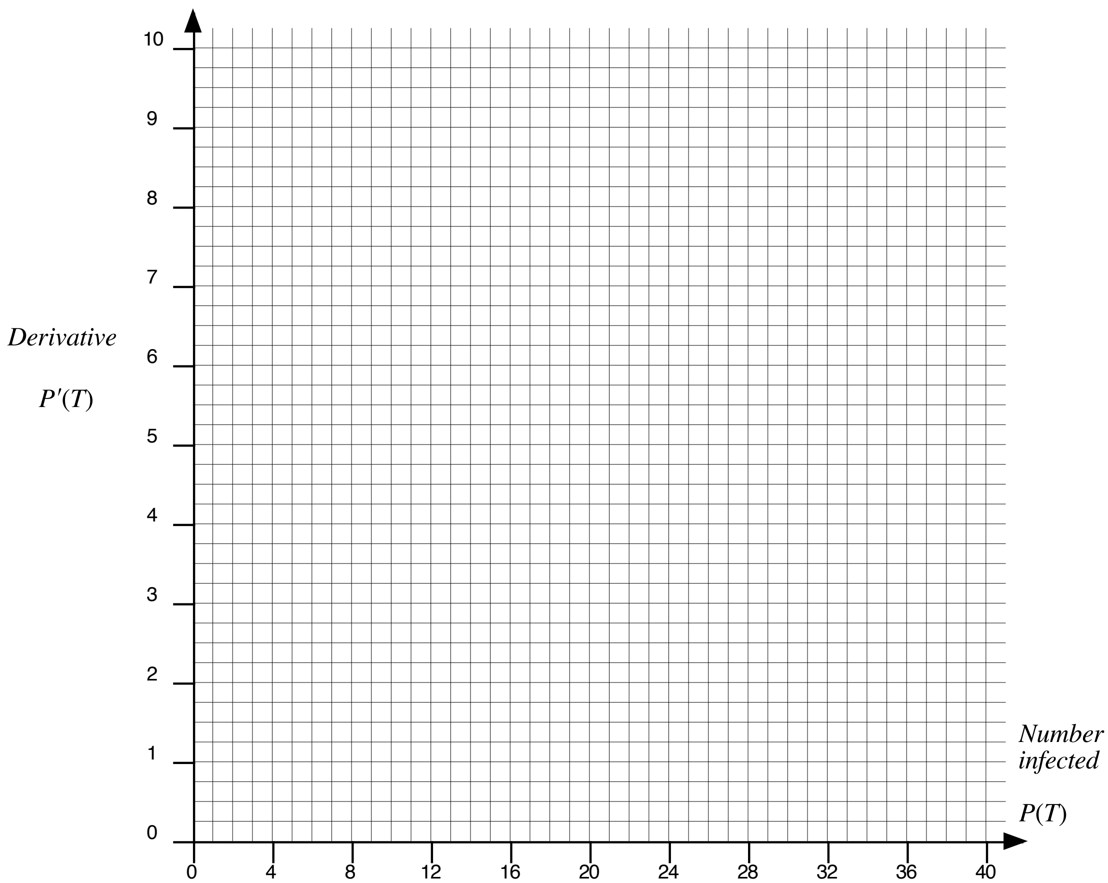
To create the differential equation you will use this approximation to get some rough values for the derivative $P'(T)$.

(c) Use the data that you recorded in Table 1 together with the difference quotient approximation to complete the entries for Table 2 (below).

Time Interval	Number of people infected at end of time interval ($P(T)$)	Derivative ($P'(T)$)
T=0 to T=1		
T=1 to T=2		
T=2 to T=3		
T=3 to T=4		
T=4 to T=5		
T=5 to T=6		
T=6 to T=7		
T=7 to T=8		
T=8 to T=9		
T=9 to T=10		
T=10 to T=11		
T=11 to T=12		
T=12 to T=13		

Table 2.

(d) Use the numbers that you recorded in Table 2 to plot a graph. When plotting your graph, use the $P(T)$ as the x-value and $P'(T)$ as the y-value.



(e) What type of function would do a reasonable job of representing the relationship between $P(T)$ (as the x-value) and $P'(T)$ (as the y-value)? Use the regression capabilities of your calculator to find an equation for this relationship.

(f) Use the equation that you found using your calculator to complete the differential equation below.

$$P'(T) =$$

Make sure that you use appropriate symbols (such as $P(T)$ and $P'(T)$ rather than x and y to write down the differential equation).