Handout 6: Calculating the Center of Mass

This handout was designed to help you to develop an understanding of what you are actually calculating when you evaluate a calculus expression like:

$$\bar{x} = \frac{\int_0^L x \cdot \delta(x)\,dx}{\int_0^L \delta(x)\,dx}.$$

You will complete this activity in teams of two or three. In order to complete the activity, you will need the following pieces of equipment:

1. A container of Play-Doh.
2. A short ruler and a long ruler.

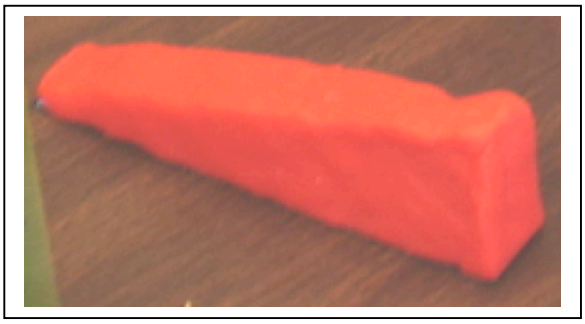
Later in the activity you will need to use a set of electronic scales, although each group of students will not need their own individual set of scales.

Part 1: Preliminary Measurements and Building a Model

The first thing that you will need to do in this activity is to find the mass of your six-inch ruler. You can measure this using the electronic scales. (Be careful to set the scale to give the mass in grams.)

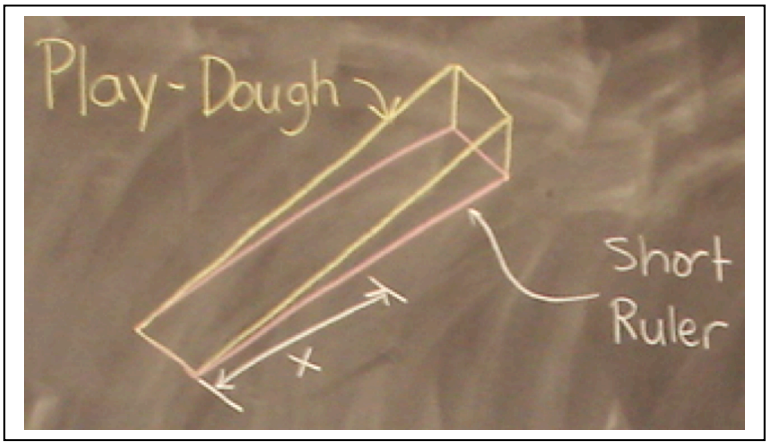
- Once you have the mass of the ruler, divide the mass by the length of the ruler (expressed in centimeters). The ratio that you get will be a constant that you will use later in the activity. This constant will be called *c*.

c = _____ grams per centimeter.



Your second task will be to build a model out of Play-Doh. Use a six-inch plastic ruler as the base of your object and build the Play-Doh model directly onto the ruler. When you create your model, do your best to create a **wedge** shape that is pointed at one end, and rises as evenly as your modeling skills allow. (See the photograph.)

Part 2: Determining the Mass of Your Model and the Constant *k*



In this activity, we will define the variable x to represent the distance from the thin end of the wedge, measured in centimeters. (See the diagram below.) With x defined in this way, the mass-density of your model is:

$\delta(x) = c + kx$ grams per cm,

where k is a positive constant and c is the constant that you obtained earlier by dividing the mass of the ruler by its length.

- Set up and evaluate an integral for the **total mass** of your Play-Doh model. When you set up your integral, pay careful attention to the limits of integration. Your final answer should contain k.

- When you have found a mathematical expression (involving k) for the total mass of your Play-Doh model, use the electronic scales to measure the total mass (in grams – make sure that the scale is set to display grams before you measure). Use this number together with the mathematical expression that you obtained earlier to work out the value of the constant k.

To find the value of k, weigh your Play-Doh model using the electronic scale and plug the reading from the scale into your mathematical expression for the total mass.

Part 3: Calculating the Center of Mass

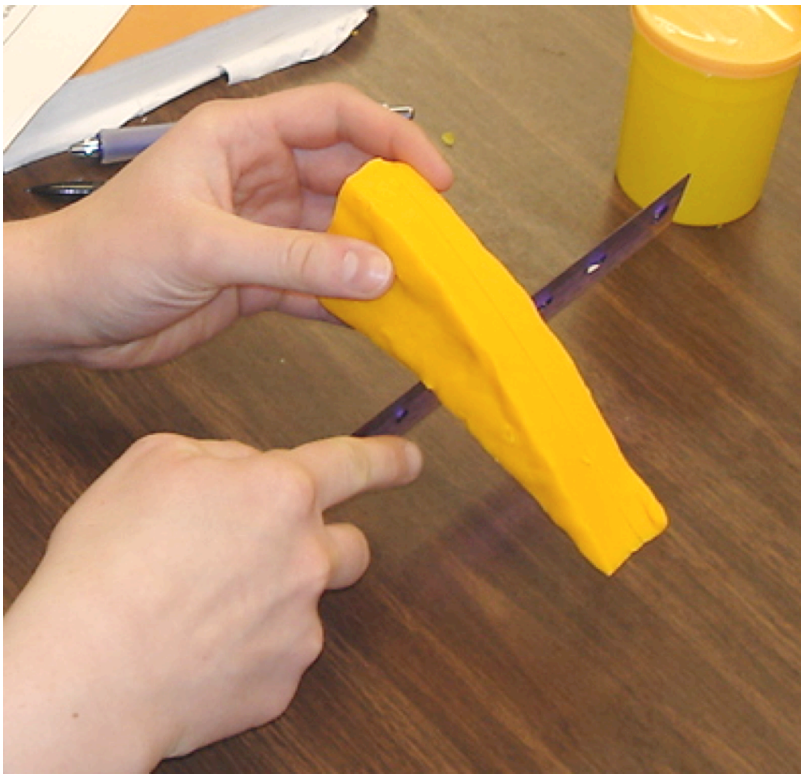
To calculate the center of mass for your Play-Doh model, you can use the formula:

$$\bar{x} = \frac{\int_0^L x \cdot \delta(x)\,dx}{\int_0^L \delta(x)\,dx},$$

where L is the length of the ruler that your Play-Doh model is built onto and $\delta(x) = c + kx$. Note that the denominator in this expression is equal to the total mass of your Play-Doh model, so instead of working this out again, you can insert the mass that you measured using the electronic scale.

Part 4: Comparing Theory and Experiment

In this last part of the activity, you should (as accurately as possible) find the balance point of your Play-Doh model. To do this, we suggest using a balance that is different from your finger, as your finger is quite wide, which will make the exact position of the balance point a little difficult to find.



To determine the exact position of the balance point, balance your model on something narrow and hard (not your finger).

- How does the x-value of the balance point of your Play-Doh model compare with the value of $\bar{x}$ that you calculated with the integral formula?