Solutions for Handout 5(a): Design of the USS Scorpion

(a) Our plan will be to model the USS Scorpion as a solid of revolution. This should provide a close approximation to the actual volume of the submarine because (as shown in Figure 2) if you were to slice up the Scorpion's hull, you would get shapes that closely resemble perfectly circular disks.

(b) The integral for the volume of the forward portion of the USS Scorpion's hull is:

$$\textit{Forward Volume} = \int_0^{28.43} \pi \cdot (4.74)^2 \cdot \left[1 - \frac{x^2}{(28.43)^2}\right] \cdot dx.$$

The integral for the volume of the aft portion of the USS Scorpion's hull is:

$$\textit{Aft Volume} = \int_{-47.09}^{0} \pi \cdot (4.74)^2 \cdot \left[1 - \frac{x^2}{(47.09)^2}\right] \cdot dx.$$

(c) Evaluating the integral for the volume of the forward portion of the USS Scorpion's hull gives:

$$\textit{Forward Volume} = \int_0^{28.43} \pi \cdot (4.74)^2 \cdot \left[1 - \frac{x^2}{(28.43)^2}\right] \cdot dx = \pi \cdot (4.74)^2 \cdot \left[x - \frac{x^3}{3 \cdot (28.43)^2}\right]_0^{28.43} = 1337.803.$$

Evaluating the integral for the volume of the aft portion of the USS Scorpion's hull gives:

$$\textit{Aft Volume} = \int_{-47.09}^{0} \pi \cdot (4.74)^2 \cdot \left[1 - \frac{x^2}{(47.09)^2}\right] \cdot dx = \pi \cdot (4.74)^2 \cdot \left[x - \frac{x^3}{3 \cdot (47.09)^2}\right]_{-47.09}^{0} = 2215.869.$$

Adding these two results together gives that the volume enclosed by the hull of the USS Scorpion was about (1337.803 + 2215.869) = 3553.672 cubic meters.

(d) The volume of air on the USS Scorpion was 65% of 3553.672 = 2309.8868 cubic meters. As one cubic meter or air has a mass of 1294.64 grams (=0.00129464 metric tons), the total mass of air in the USS Scorpion was (2309.8868)×(0.00129464) = 2.99047 metric tons. The mass of the hull, equipment and fixtures of the USS Scorpion was about 2790 metric tons, so the total mass of the submarine was about (2790 + 2.99047) = 2792.99047 metric tons.

(e) The mass of the sea water that had the same volume as the USS Scorpion was (3553.672) ×(1.0278) = 3652.46408 metric tons. Using Archimedes' Principle, the buoyant force experienced by the USS Scorpion would have been:

$$\textit{Buoyant Force} = 3652.46408 - 2792.99047 = 859.4736 \text{ metric tons.}$$

(f) One cubic meter of air has a mass of 0.0012464 metric tons, and each cubic meter of sea water has a mass of 1.0278 metric tons. Therefore, every cubic meter of air expelled by the submarine and replaced by seawater will reduce the buoyant force by:

$$1.0278 - 0.0012464 = 1.0265536 \text{ metric tons.}$$

Therefore, the number of cubic meters of air that should be expelled and replaced by seawater to create neutral buoyancy for the USS Scorpion was:

$$\frac{859.4736}{1.0265536} = 837.2418 \approx 837 \text{ cubic meters.}$$

So, roughly 25% of the air spaces within the submarine had to be flooded with water in order for the submarine to gain neutral buoyancy.