Handout 4(b): Improper Integrals through Algebraic Comparisons

For each of the integrals given on this handout, determine whether the integral converges or diverges. You should not use your calculator to evaluate antiderivatives here. If you can't work out an antiderivative, try comparing the integral in question to an improper integral that is easier to work out (e.g. a *p*-integral or the integral of an exponential function).

If you do use comparison to work out the convergence or divergence of any of the improper integrals, note down:

(I) Your initial guess concerning convergence or divergence.

(II) The improper integral that you plan to use for comparison.

(III) The work that shows that the integral from Step (II) is greater than or less than the integral you are investigating.

(IV) Your final conclusion.

(a) $$\int_3^{\infty} \frac{1}{1+x^{\frac{3}{2}}}\,dx.$$

Initial Guess: Converges Diverges

Improper Integral for Comparison:

(b) $$\int_4^{\infty} \frac{3+\sin(x)}{x}\,dx.$$

<u>Initial Guess:</u> Converges Diverges

<u>Improper Integral for Comparison:</u>

(c) $$\int_{2}^{\infty} \frac{1}{x^2 + 5x + 1}\,dx.$$

Initial Guess: Converges Diverges

Improper Integral for Comparison:

(d) $$\int_{2}^{\infty} x \cdot e^{-x} \cdot dx.$$

Initial Guess: Converges Diverges

Improper Integral for Comparison:

Answers:

(a) Converges. Good comparison is to $\int_{3}^{\infty} x^{-\frac{3}{2}} dx$.

(b) Diverges. Good comparison is to $\int_{4}^{\infty} \frac{2}{x} dx$.

(c) Converges. You can calculate this directly but it is easier to compare; a good comparison is to $\int_{2}^{\infty} \frac{1}{x^2} dx$.

(d) Converges. You can calculate this directly using integration by parts or compare; a good comparison is to $\int_{2}^{\infty} e^{-x/2} dx$.