

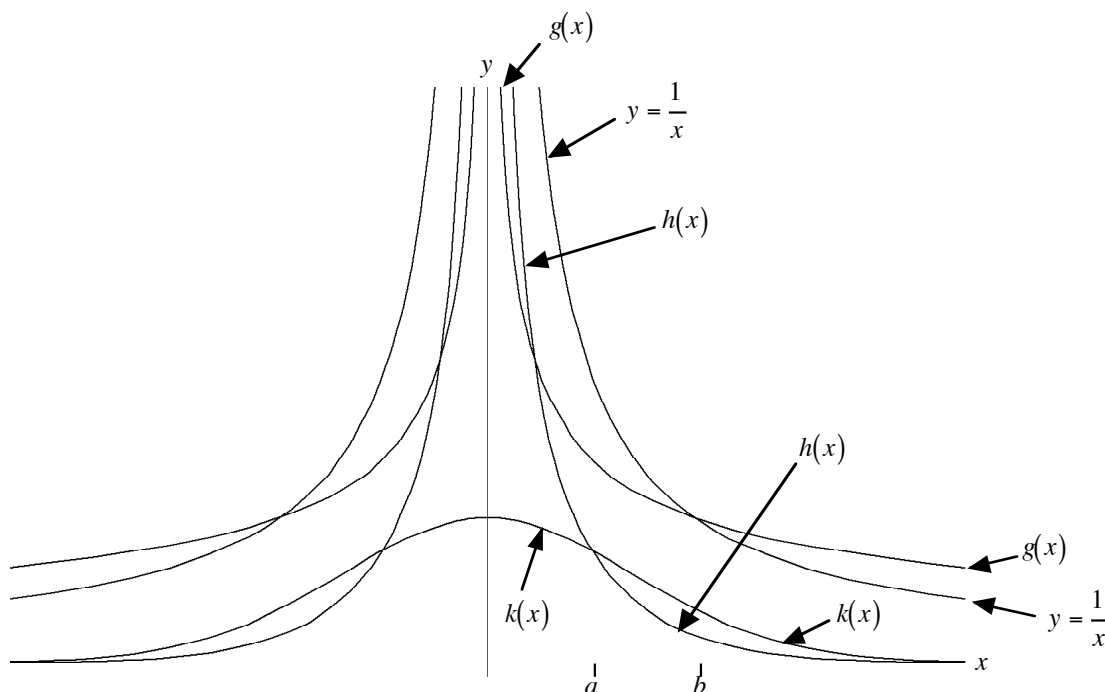
Handout 4(a): Improper Integrals through Graphical Comparisons

The diagram given below shows the functions $g(x)$, $h(x)$, and $k(x)$. You can assume that the domain of each function is either all real numbers or all real numbers except $x = 0$. In this problem a and b are positive real numbers that satisfy $0 < a < b < \infty$. You may assume that when $x > b$, $h(x) < k(x) < \frac{1}{x} < g(x)$ and when $0 < x < a$, $k(x) < h(x) < \frac{1}{x}$. In addition, you may assume that:

$$\int_0^{\infty} k(x) dx \text{ converges.}$$

$$\int_0^a g(x) dx \text{ converges.}$$

$$\int_a^{\infty} g(x) dx \text{ diverges.}$$



Determine whether each of the integrals listed below converges, diverges, or if more information is needed to say for sure. Circle ONE response ("converges," "diverges" or "need more information") for each integral.

- (a) $\int_0^a h(x) dx$ **CONVERGES** **DIVERGES** **NEED MORE INFORMATION**

Reasoning:

(b)	$\int_a^{\infty} h(x) dx$	CONVERGES	DIVERGES	NEED MORE INFORMATION
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Reasoning:

(c)	$\int_b^{\infty} h(x) dx$	CONVERGES	DIVERGES	NEED MORE INFORMATION
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Reasoning:

(d)	$\int_0^a x \cdot h(x) dx$	CONVERGES	DIVERGES	NEED MORE INFORMATION
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Reasoning: