Recitation Handout 2(b): Trigonometric Integrals

Evaluate each of the integrals given in the table below. In each case, describe the "trick" that is used to convert the integral into something that you can handle more easily.

Indefinite integral	***Work and equation for antiderivative***
(a) $\int x\sin^2(x)dx$	
Trick	Use double angle formula: $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$ to produce two integrals (one easy; one possible via integration by parts)
(b) $\int_0^{\frac{\pi}{2}}\sin^3(x)\cos^6(x)dx$	
Trick	Use identity: $\sin^2(x)+\cos^2(x)=1$ to create a single factor of sine or cosine (that will allow integration by reversing the Chain Rule)

(c) $\int \tan(x)\sec^6(x)dx$	
Trick	Re-write the integral as $\tan(x)\sec(x)$ times $\sec^5(x)$ to facilitate u-substitution.
(d) $\int x\tan(x^2)dx$	
Trick	Use the integration formula: $\int \tan(x)dx = \ln(\lvert\sec(x)\rvert) + C$

(e) $\int \frac{e^x}{\cos(e^x)}\,dx$	
Trick	Use the integration formula: $\int \sec(x)dx = \ln(\lvert\sec(x)+\tan(x)\rvert)+C$
(f) $\int_{\frac{\pi}{2}}^{\pi} \cos^2(x)dx$	
Trick	Use double angle formula: $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

(g) $\int \sin(4x)\cos(3x)dx$	
Trick	Use angle addition formulas: $\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)$ $\sin(a-b)=\sin(a)\cos(b)-\cos(a)\sin(b)$ to express the product as a sum of sin(7x) and sin(x).
(h) $\int \cos^4(x)\sin(2x)dx$	
Trick	Use the double angle formula: $\sin(2x)=2\sin(x)\cos(x)$ to turn put this entirely in terms of sin(x) and cos(x) which can be integrated by substitution.

(i) $\int \tan^3(x)\sec^5(x)dx$	
Trick	Use identity: $\tan^2(x)+1=\sec^2(x)$ to express $\tan^2(x)$ in terms of $\sec^2(x)$, leaving a factor of $\sec(x)\tan(x)$ to accompany powers of $\sec(x)$

Answers

(a) $\frac{1}{4}x^2-\frac{1}{8}\cos(2x)-\frac{1}{4}x\sin(2x)+C$

(b) $\frac{2}{63}$

(c) $\frac{1}{6}\sec^6(x)+C$

(d) $\frac{1}{2}\ln(|\sec(x^2)|)+C$

(e) $\ln(|\sec(e^x)+\tan(e^x)|)+C$

(f) $\frac{\pi}{4}$

(g) $\frac{-1}{14}\cos(7x)-\frac{1}{2}\cos(x)+C$

(h) $\frac{-1}{3}\cos^6(x)+C$

(i) $\frac{1}{7}\sec^7(x)-\frac{1}{5}\sec^5(x)+C$