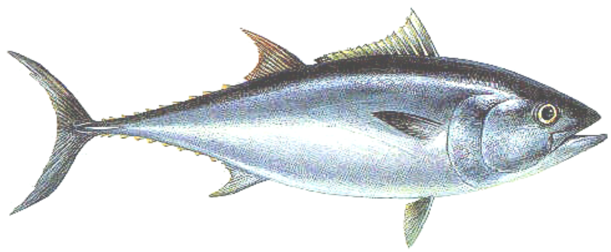
Recitation Handout 16: Taylor Series and the Atlantic Bluefin Tuna

The giant bluefin tuna (*Thunnus thynnus*) is the largest bony fish known to science. This fish can grow to a length of eleven feet and weigh up to 1500 pounds[1]. The bluefin tuna is a remarkably strong fish, and is able to retract its fins and eyes to make it more streamlined. Bluefin tuna have been observed to swim at speeds of up to 55 miles per hour[2].



Bluefin tuna are valued as a food source, especially as sushi and sashimi. A large tuna in excellent condition sell for more than $100,000 when auctioned at *Tsukiji*, the main fish market in Tokyo, Japan[3]. Bluefin have been commercially fished in the western Atlantic since the 1960's, with the industry firmly established by the 1980's[4]. Bluefin fishing has become such a lucrative business that commercial tuna fishermen routinely use spotter aircraft to find the fish[5].

Contrary to many perceptions[6], most tuna fisheries, although heavily exploited are not seriously over fished[7]. This is not the case for bluefin tuna. Bluefin have been heavily overf ished for at least two decades[8]. Based on studies conducted by the International Convention for the Conservation of Atlantic Tuna (ICCAT) and the National Research Council (NRC), the breeding population of Atlantic bluefin tuna fell from approximately 235,000 in 1975 to less than 40,000 in the late 1990's.

In 1998, ICCAT proposed an historic plan to limit catch sizes of bluefin tuna to allow the population to recover. In this handout, you investigate the effects of the ICCAT plan on the population of Atlantic bluefin tuna.

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[1] Source: New England Aquarium (www.neaq.org). The largest bluefin tuna on record was caught by Ken Fraser in Canada during 1979. The fish that Fraser caught weighed 677 kg (1497 pounds).
[2] Source: World Wildlife Fund. (www.panda.org).
[3] Source: NASA and the Smithsonian Institution Ocean Planet Project (seawifs.gsfc.nasa.gov).
[4] Source: National Academy of Sciences, National Research Council. *An Assessment of Atlantic Bluefin Tuna*. Washington, DC: National Academy Press, 1994.
[5] Source: World Wildlife Fund.
[6] For example, see: Cole, J. N. "The Vanishing Tuna." *Atlantic Monthly*, Volume 239, p. 50. (Dec. 1976)
[7] Source: Environmental Protection Agency, Revised Final Enviromental Impact Statement to accompany Fisheries Management Plan for Highly Migratory Species, 1999.
[8] Source: Buck, Eugene. "Atlantic Bluefin Tuna: International Management of a Shared Resource." The National Council for Science and the Environment, Washington DC, 1995.

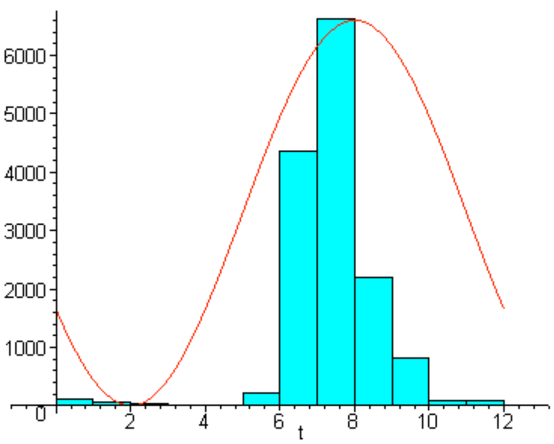

Figure 1: Number of bluefin tuna caught by month, 1998.

The commercial bluefin fishing season runs from June 1 to May 31, or until the quota has been reached[9]. As shown in the histogram (Figure 1, left) most fish are caught between July and October, with relatively few fish harvested during the rest of the year[10]. Regulations governing the size of tuna that can be caught[11] mean that it is mainly sexually mature adults that are harvested.

The curve in Figure 1 is a rough model for the rate at which tuna caught (in tuna caught per month)[12]. The equation for the curve is:

$$Rate = 3.3 - 3.3\cos\left(\tfrac{\pi}{6}(t-2)\right)$$

where the rate of capture has the units of thousands of tuna per month, and t represents the number of months since January 1998 (when the ICCAT plan went into effect).

Studies are now underway to study the bluefin tuna, and particularly whether or not the plan proposed by ICCAT in 1991 will enable the bluefin tuna population to recover or not. These studies involve attaching electronic tags to tuna. These tags are monitored by satellites and the information collected about the tuna relayed to scientists in the US and Canada[13]. Some of the data collected from these studies is shown in Figure 2. The rates of change given in Figure 2 are for sexually mature bluefin tuna and do not include juveniles.

The information given in Figure 2 (see next page) has been synthesized into a mathematical model for the Atlantic bluefin tuna population. This model can be represented by a differential equation and an initial condition:

$$\frac{dP}{dt} = 0.0875 \cdot P(t) - 3.3 + 3.3 \cdot \cos\left(\tfrac{\pi}{6}(t-2)\right) \qquad \text{and} \qquad P(0) = 40,$$

where $P(t)$ is the size of the sexually mature Atlantic bluefin tuna population (in units of thousands of tuna) and t is the number of months since January 1998.

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[9] Source: U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Marine Fisheries Service. "Small Entity Compliance Guide for the Consolidated Regulations of Atlantic Tuna, Swordfish, Sharks and Billfish." 1999.

[10] Source of data: Personal communication from the National Marine Fisheries Service, Fisheries Statistics and Economics Division, Silver Spring, MD.

[11] Source: U.S. Department of Commerce, National Oceanic and Atmospheric Administration.

[12] Source of data: National Marine Fisheries Service, Fisheries Statistics and Economics Division, and Roberta Holland. "Bluefin tuna prices hit rock bottom." *Boston Business Journal* (September 11, 1998).

[13] Some of this research is being carried out locally by Dr. Molly Lutcavage a marine biologist at the New England Aquarium.

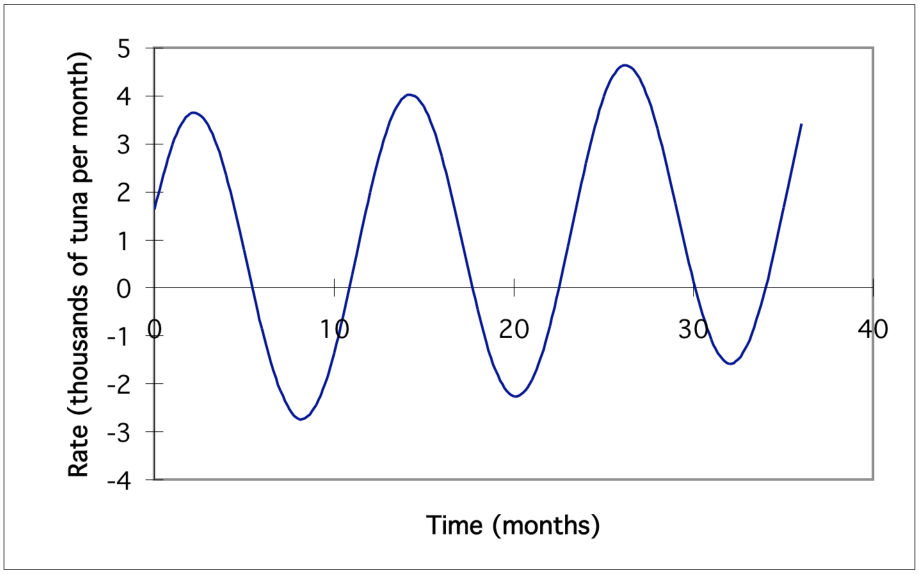
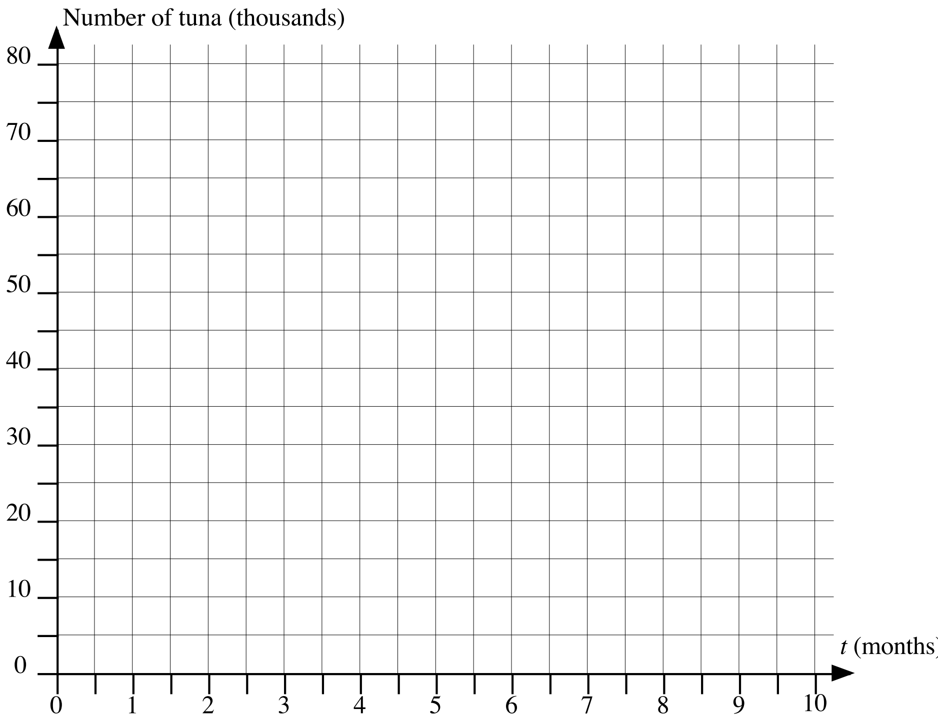


Figure 2: Rate of change of bluefin tuna breeding population (thousands of tuna per month) 1998-2001. (Source: International Commission for Conservation of Atlantic Tuna).

1. Use the axes provided below, the information that $P(0) = 40$ and Figure 2 to sketch a plausible graph for the function $P(t)$. Based on your sketch, did the ICCAT help the Atlantic bluefin tuna population to increase their numbers?



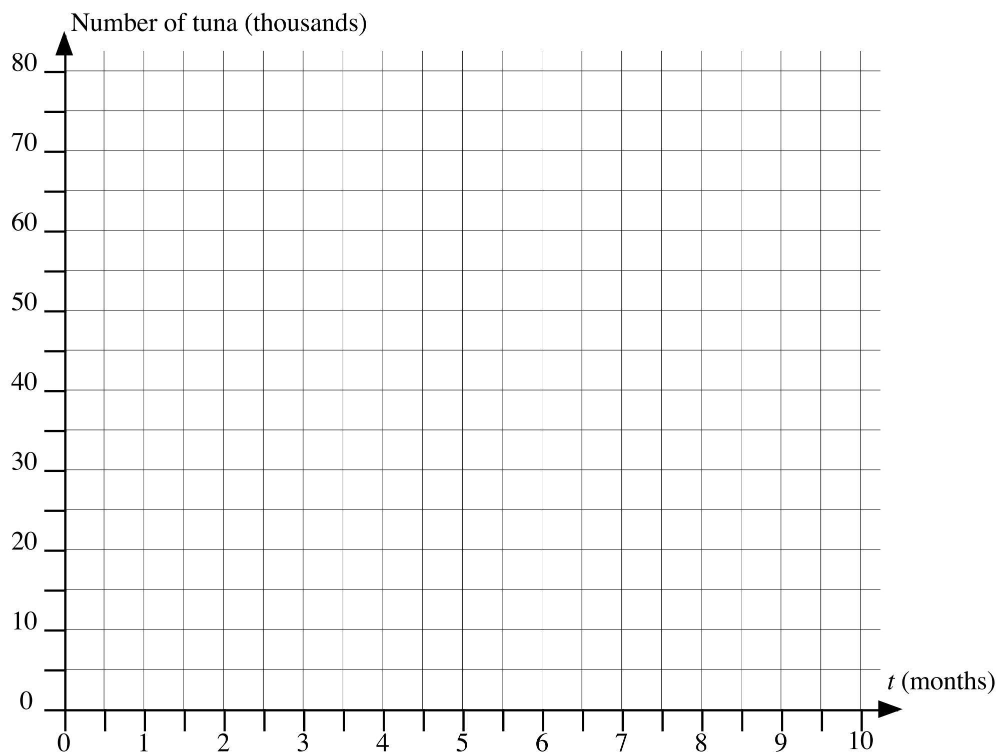
2. Use the differential equation and initial condition to find the formula for the tangent line approximation to the curve $y = P(t)$ that is based at $t = 0$. What does the tangent line suggest about the effectiveness of the ICCAT plan that was intended to help the Atlantic bluefin tuna?

3. Using the same set of axes that you used in Question 1, sketch the tangent line. For which values of t does the tangent line seem to do a decent job of approximating the function $P(t)$?

4. Find the formula for the second degree Taylor polynomial approximation of the function $P(t)$ that is based at zero. (That is, the anchor point is $a = 0$.)

5. Find the formula for the degree three Taylor polynomial approximation of the function $P(t)$ that is based at zero. (That is, the anchor point is $a = 0$.)

6. Use the axes given below to sketch your version of the $P(t)$ function from Question 1. Then graph your third degree Taylor polynomial on your calculator and transfer the graph to the set of axes given below. Based on your sketch, over what set of t-values does the Taylor polynomial do a decent job of matching the graph of $P(t)$?



7. Why do you think a marine biologist or fisheries researcher might want to create a Taylor polynomial to approximate the function $P(t)$? What would be the main advantage of using a Taylor polynomial with a lot of terms in it?