Solution for Recitation Handout 13: Maximum Safe Dose of Prozac

Let D represent the daily dose of Prozac® in mg.

The strategy that we will follow to solve this problem is:

- **a.** Find an exponential function to give the mg of Prozac® that remain in a a person's body from a single dose of the drug.
- **b.** Use the exponential function to create a table that shows how the amount of Prozac® in the body builds up with repeated, daily doses of the drug.
- **c.** Extrapolate the expressions in the table to create a series that will give the peak amount of Prozac® in the person's body after they have taken N doses in addition to the initial dose.
- **d.** Examine what happens to the sum of the series as $N \to \infty$.

Step (a).

Let T represent the time in units of days.

The half-life of fluoxedine is five days. Therefore, if the amount of fluoxedine at time $T = 0$ is D, then the amount of fluoxedine at time $T = 5$ will be $0.5 \cdot D$.

Using these to calculate a formula for an exponential function, $y = A \cdot B^T$ by hand gives the following formula.

$$\text{mg of fluoxedine from a single dose of Prozac®} = D \cdot (0.87055)^T.$$

Step (b)

Assume that the patient takes a dose of Prozac once per day, and that each dose delivers exactly D mg of fluoxedine.

Number of days on Prozac®	Maximum mg of fluoxedine in body
1	D
2	$D + D \cdot (0.87055)$
3	$D + D \cdot (0.87055) + D \cdot (0.87055)^2$
4	$D + D \cdot (0.87055) + D \cdot (0.87055)^2 + D \cdot (0.87055)^3$

Step (c)

After N days of taking Prozac®, the maximum mg of fluoxedine in the person's body will be:

$$D + D \cdot (0.87055) + D \cdot (0.87055)^2 + ... + D \cdot (0.87055)^k + ... + D \cdot (0.87055)^{N-1}.$$

This is a geometric series with:

- **Initial value:** $a = D$.
- **Multiplicative factor:** $r = 0.87055$.
- **Number of terms added:** N.

The sum of this series will be given by the geometric summation formula:

$$\text{Maximum mg of fluoxedine} = \frac{D \cdot (1 - 0.87055^N)}{1 - 0.87055}.$$

Step (d)

If the patient keeps taking Prozac® for a very long time (e.g. ten years) then N will get very, very big. As the number, 0.87055, that is raised to the power of N is less than one, 0.87055^N will get closer and closer to zero as N gets really, really big. Therefore, when N is big,

$$\text{Maximum mg of fluoxedine} = \frac{D \cdot (1 - 0.87055^N)}{1 - 0.87055} \approx \frac{D \cdot (1 - 0)}{1 - 0.87055} = \frac{D}{1 - 0.87055}.$$

The lowest amount of fluoxedine that is suspected to have caused a death due to poisoning is 760 mg. Therefore, you would want the maximum mg of fluoxedine to remain below this level. What value of D will keep the maximum mg of fluoxedine under 760 mg?

$$\frac{D}{1 - 0.87055} = 760.$$

Solving this for D gives $D = 98.4$ mg.

Prozac® is dispensed (see Figure 6 of the handout) in 20 mg capsules so the maximum safe dose will be the greatest multiple of 20 mg that is less than $D = 98.4$ mg. This is 80 mg, just as stated in the Physicians' Desk Reference.