Handout 7: How much D'oh! is in a do-nut?

Figure 1: Homer J. Simpson.

In an episode[1] of the popular television program, "The Simpsons," Homer Simpson (see Figure 1[2]) became the conductor of Springfield's monorail. The episode began with Homer leaving his job at the nuclear power plant singing:

Homer, Homer Simpson, He's the greatest guy in history.
From the town of Springfield, He's about to hit a chestnut tree.

After Homer's car crash, Mr. Burns and Smithers were caught dumping toxic waste from the nuclear power plant. Mr. Burns was fined three million dollars. The people of Spingfield met to discuss how to spend this windfall, when fast-talking music man Lyle Lanley persuaded them to spend it all on a monorail.

Lyle Lanley: Well, sir, there's nothing on Earth like a genuine, bona fide, electrified, six-car monorail! What'd I say?
Ned Flanders: Monorail!
Lyle Lanley: What's it called?
Patty and Selma: Monorail!
Lyle Lanley: That's right – Monorail!
(Crowd softly chants "Monorail" in rhythm.)
Miss Hoover: I hear those things are awfully loud.
Lyle Lanley: It glides as softly as a cloud.
Apu: Is there a chance the track could bend?
Lyle Lanley: Not on your life, my Hindu friend.
Barney: What about us brain-dead slobs?
Lyle Lanley: You'll all be given cushy jobs.
Abraham Simpson (Grandpa): Were you sent here by the devil?
Lyle Lanley: No, good sir, I'm on the level.
Chief Wiggum: The ring came off my pudding can.
Lyle Lanley: Use my pen knife, my good man. I swear it's Springfield's only choice! Throw up your hands and raise your voice!
Everyone: Monorail!
Lyle Lanley: What's it called?
Everyone: Monorail!
Lyle Lanley: Once again!
Everyone: Monorail!
Marge: But Main Street's still all cracked and broken!
Bart: Sorry, Mom, the mob has spoken.
Everyone: Monorail! Monorail! Monorail! Monorail! Monorail! (Finishing with a tuneful flourish.)
Homer: Mono…D'oh!

Figure 2: Mascot and road sign for the "Lard Lad" chain of donut restaurants.

[1] Episode 9F10 "Marge versus the Monorail" to be precise. This episode was written by Harvard alumnus Conan O'Brien and guest starred Leonard Nimoy as himself. Nimoy spoke at the monorail dedication ceremony commenting, "I'd say that this vessel could do at least warp five," to which Mayor Quimby added, "And, ah, may the force be with you."
[2] Image source: http://www.lardlad.com/

Built to remarkably poor standards, the monorail went out of control on its maiden voyage. Homer finally stopped the runaway train by anchoring the monorail to the giant donut outside of a "Lard Lad" donut shop (see Figure 2[3]).

In this handout, you will set up a mathematical expression for the volume of a donut and approximate the value of the integral using your calculator.

You can imagine the donut (mathematically, a solid torus) being formed by revolving the shaded disk shown in Figure 3[4] in a big circle around the y-axis.

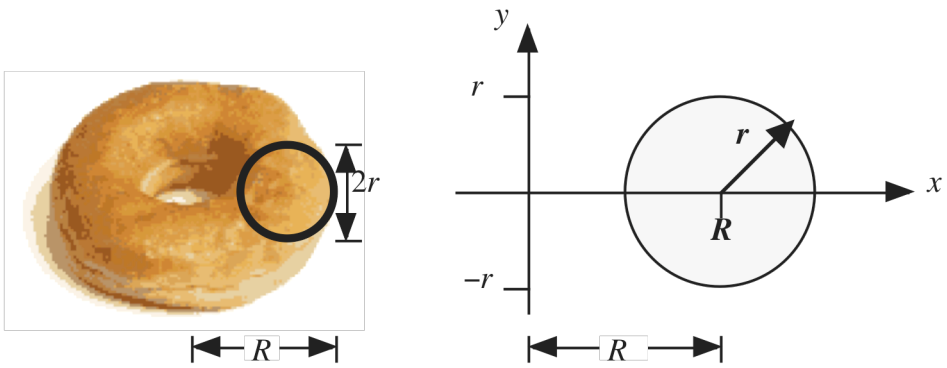

Figure 3: Mathematical model of a donut (a solid torus).

1. Working from a physical example of a donut, measure the values of the constants r and R.

2. With the aid of a diagram, show how you can break the volume of a donut up into convenient pieces whose volume may be approximated using straightforward geometry.

[3] Image source: http://www.lardlad.com/
[4] Image source: http://home.earthlink.net/~jiw/donuttypes.html

3. Figure 4 shows a cylindrical shape. Create a formula for the volume of this shape – that is, a formula for the gray shape. Apart from constants, your formula should only involve the symbols x, dx and $f(x)$.

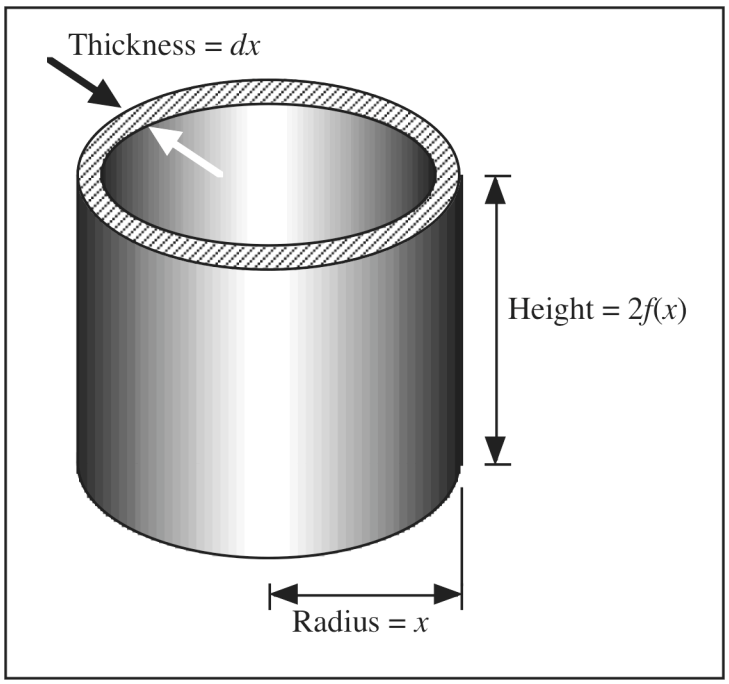

Figure 4: The cylindrical shape.

4. Use the volume formula that you have found to set up an integral (including upper and lower limits of integration) whose value will be equal to the volume of one standard-sized donut.

5. Use the integral that you have formulated (along with the "sum" and "seq" capabilities of your calculator if you can't find an anti-derivative) to find the volume of a standard sized donut.

Epilogue

Just in case you think that this is a silly calculation whose only practical application is to provide pointless busy-work to Math 122 students, note that calculations much like this one lie at the heart of research into generating electrical power by means of nuclear fusion[5]. Figures 5[6] and 6[7] show the interiors of the "Tokamak" (located in Princeton, NJ) and "Joint European Tourus" (located in Abingdon, Oxfordshire, England). The volumes of these spaces had to be exquisitely accurately calculated, and they are shaped exactly like donuts.

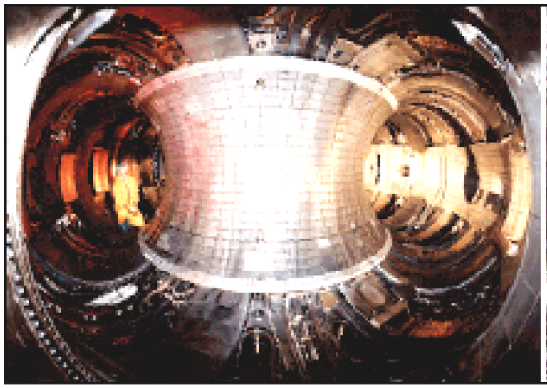

Figure 5: The interior of the Tokamak Fusion Test Reactor, Princeton Plasma Physics Laboratory.

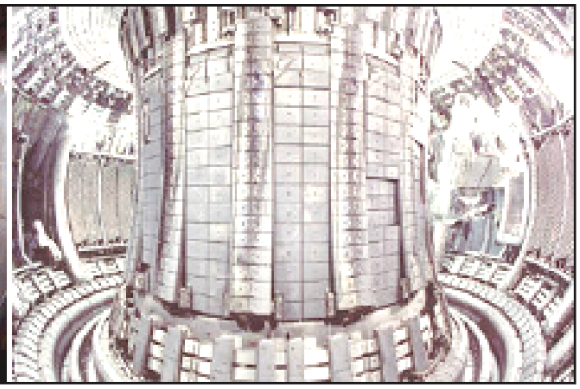

Figure 6: The interior of the Joint European Torus, the main research facility of the European Fusion Development Agreement.

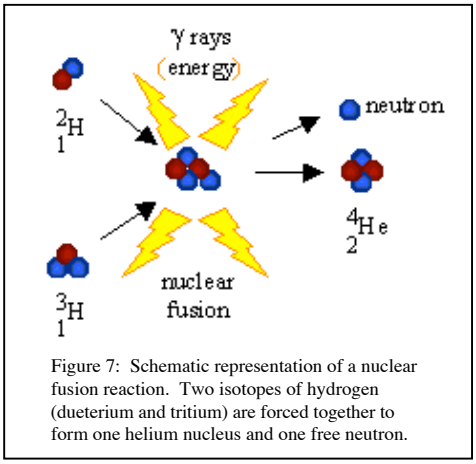

Figure 7: Schematic representation of a nuclear fusion reaction. Two isotopes of hydrogen (dueterium and tritium) are forced together to form one helium nucleus and one free neutron.

Nuclear fusion (see Figure 7[8]) is a process in which two small, light nuclei (such as the isotopes ^{2}H, deuterium, and ^{3}H, tritium, of hydrogen) are forced together under conditions of extreme heat and pressure to form a single, heavier nucleus (such as helium, ^{4}He). The combined mass of the two small nuclei is slightly less than the mass of the helium nucleus formed. The small difference in mass is converted to energy (with the amount of energy given by Einstein's famous equation $E = m \cdot c^2$). It is this energy that scientists and engineers hope to harness as an alternative energy source[9] in the future.

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[5] At 7:44pm on Saturday 9 November 1991, the Joint European Torus (JET) generated power by means of a nuclear fusion reaction for the first time in human history. The JET generated between 1.5 and 2 megawatts of power. In order to do so, the researchers had to heat a mixture of deuterium and tritium gas to about 200 million degrees Celsius. This is about 10 times hotter than the interior of the Sun. (Source: Joint European Torus Project Press Release, 1991.)

[6] Image source: http://www.physics.auburn.edu/~plasma/fusion/fusion_lab/fusion.htm

[7] Image source: http://www.fusion.org.uk/culham/jetmain.html

[8] Image source: http://www.visionlearning.com/

[9] The fuel needed for running a fusion reactor is hydrogen, which is incredibly abundant on the Earth. The oceans, for example, are vast reservoirs of hydrogen in the form of water (H_2O).