Handout 13: Using Convergence Tests

Determine the convergence or divergence of each of the following series. In each case, demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test*. Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work and how the convergence test justifies your answer.

(a) Does the following series converge or diverge? $\displaystyle\sum_{n=1}^{\infty} \frac{12(n+1)^2}{4n^2-2n+1}$

CONVERGENCE TEST USED:

STEP-BY-STEP JUSTIFICATION:

FINAL CONCLUSION: CONVERGES DIVERGES

(b) Does the following series converge or diverge? $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot \sqrt{n}}{n+1}$

CONVERGENCE TEST USED:

STEP-BY-STEP JUSTIFICATION:

FINAL CONCLUSION: CONVERGES DIVERGES

(c) Does the following series converge or diverge? $\sum_{n=1}^{\infty} \frac{n}{e^n}$

CONVERGENCE TEST USED:

STEP-BY-STEP JUSTIFICATION:

FINAL CONCLUSION: CONVERGES DIVERGES

(d) Does the following series converge or diverge? $\sum_{n=1}^{\infty} \frac{(n!)^2}{1000^n}$

CONVERGENCE TEST USED:

STEP-BY-STEP JUSTIFICATION:

FINAL CONCLUSION: CONVERGES DIVERGES

(d) Does the following series converge or diverge? $\sum_{n=1}^{\infty} \frac{n-3}{n^3+16}$

CONVERGENCE TEST USED:

STEP-BY-STEP JUSTIFICATION:

FINAL CONCLUSION: CONVERGES DIVERGES

Overall Game Plan for Using Convergence Tests:

When you first see the series, try:

If that doesn't work and you notice that the terms of the series alternate between + and −, try:

If the first test didn't and the series is not alternating, next try:

- **Use this test also when:**

- **When using this test, remember to:**

If you get a formula for $f(x)$ that is too hard to integrate, try:

- **Also use this when:**

If all else has failed, try:

- **When doing this, remember the convergence results for *p*-series:**

ANSWERS:
(a) Diverge. **(b)** Converge. **(c)** Converge. **(d)** Diverge.
(e) Converge.