

MATH 122 – SECOND UNIT TEST

Friday, October 24, 2008.

NAME: _____

Sona
Akopian

A

Brian
Seguin

B

C

Paul
McKenney

D

E

Oleksii
Mostovyi

F

G

Jason
Rute

H

I

Lisa
Espig

J

K

Instructions:

1. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
2. Please read the instructions for each individual question carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated. However, if you are asked to find an **exact** value of a quantity that involves an integral then you should not use calculator integration for this.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. If you evaluate an improper integral, be sure to use appropriate algebraic and limit notation.
7. Please **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

Problem	Total	Score
1	20	
2	15	
3	20	
4	15	
5	15	
6	15	
Total	100	

1. 20 Points. SHOW ALL WORK. NO WORK = NO CREDIT.

Solve each of the following initial value problems. Your final answers should not include any unspecified constants. Clearly indicate your final answer, for example by circling it.

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

(a) (10 points) $\frac{dP}{dt} = 10 - 0.5 \cdot P$ $P(0) = 15$

Continued on the next page.

Solve each of the following initial value problems. Your final answers should not include any unspecified constants. Clearly indicate your final answer, for example by circling it.

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

(b) (10 points) $\frac{dz}{dt} + \cos(t) \cdot z = \cos(t)$ $z(0) = 3$

2. 15 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.

You should not use your calculator on this problem for anything besides arithmetic. In particular, finding antiderivatives or evaluating improper integrals on your calculator is not acceptable.

Calculate the **exact** volume of the solid created by revolving the region of the xy -plane bounded by the curves:

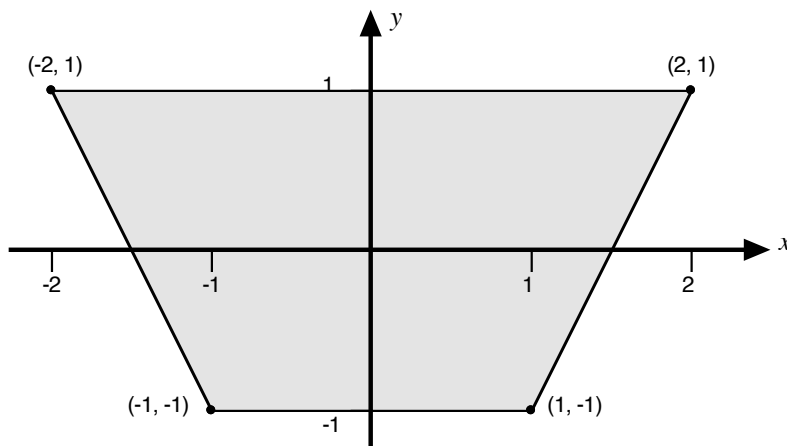
$$x = \frac{3\pi}{2} \qquad x = 2\pi \qquad y = 0 \qquad y = \cos(x)$$

around the y -axis. Clearly indicate your final answer.

3. 20 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

The object that you will be studying in this problem is a trapezoidal plate with the dimensions shown in the diagram given below. All dimensions and measurements are in units of meters.



- (a) **(10 points)** The density of the trapezoidal plate varies from place to place. The density of the plate at any point is given by the function:

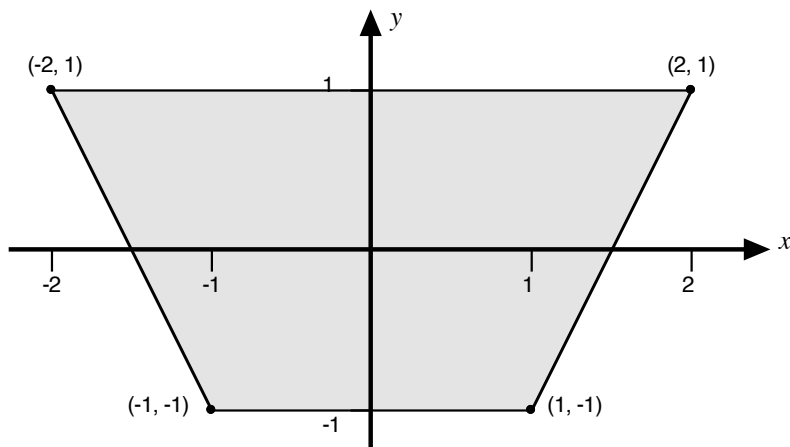
$$\delta(y) = 2 + y \text{ kg/m}^2.$$

Find the **exact** total mass of the trapezoidal plate. Clearly indicate your final answer.

Continued on the next page.

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

The object that you will be studying in this problem is a trapezoidal plate with the dimensions shown in the diagram given below. All dimensions and measurements are in units of meters.



- (b) **(10 points)** The x -coordinate of the trapezoidal plate's center of mass is $x = 0$. (You do not have to show this.) Find the **exact** y -coordinate of the trapezoidal plate's center of mass. Clearly indicate your final answer.

4. 15 Points. SHOW YOUR WORK. EXPLAIN IN PART (b).

The function $y = f(x)$ is defined by the following differential equation and initial condition:

Differential equation: $f'(x) = x + f(x)$ or $y' = x + y$

Initial value: $f(0) = 1$ or $y(0) = 1$.

(a) (10 points) Use Euler's Method and $\Delta x = 0.5$ to estimate $f(2.5)$. Clearly indicate your final answer.

(b) (5 points) Is the estimate of $f(2.5)$ that you calculated in Part (a) an over-estimate or an under-estimate of the actual value of $f(2.5)$? Be careful to show your work and explain how you know.

5. 15 Points. SHOW ALL WORK. NO WORK = NO CREDIT.

Find the solution to the following initial value problem. Note that your final answer should not contain any unspecified constants. Clearly indicate your final answer.

$$y'' + 9 \cdot y = e^x$$

$$y(0) = 1.1$$

$$y'(0) = 6.1$$

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

Additional space is provided on the following page if you need it.

FINAL ANSWER: _____

Find the solution to the following initial value problem. Note that your final answer should not contain any unspecified constants. Clearly indicate your final answer.

$$y'' + 9 \cdot y = e^x$$

$$y(0) = 1.1$$

$$y'(0) = 6.1$$

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

FINAL ANSWER: _____

6. 15 Points. CLEARLY INDICATE YOUR FINAL ANSWER.

A cylindrical urn, standing upright, contains hot chocolate. The top of the urn has a *diameter* of 0.8 m. The urn has a *height* of 1.1 m and is filled to a *depth* of 0.9 m. The density of the hot chocolate at a depth of h meters below the surface is given by the function:

$$\delta(h) = 1 + A \cdot h \quad \text{kg/m}^3,$$

where A is a positive constant.

Set up an integral that will give the total work (in joules) that must be done in order to pump the hot chocolate to the top rim of the urn. Remember to include appropriate units in your answer.

- NOTE:**
- (1) Your final answer may include the constant A .
 - (2) In metric (or SI) units, the constant for gravity is $g = 9.8 \text{ m/s}^2$.
 - (3) You do not have to evaluate the integral.