MATH 122 – THIRD UNIT TEST

Thursday, December 4, 2008.

NAME: ____

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A	B	D	F	H	J
	C	E	G	I	K

Instructions:

1. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
2. Please read the instructions for each individual question carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work for each exam question so that graders can see your final answer **and** how you obtained it.
4. You may use your calculator on all exam questions except where otherwise indicated. However, if you are asked to find an ***exact*** value of a quantity that involves an integral then you should not use calculator integration for this.
5. If you use graphs or tables to obtain an answer (especially if you create the graphs or tables on your calculator), be certain to provide an explanation and a sketch of the graph to show how you obtained your answer.
6. If you evaluate an improper integral, be sure to use appropriate algebraic and limit notation.
7. Please **TURN OFF** all cell phones and pagers, and **REMOVE** all headphones.

Problem	Total	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

1. 20 Points. SHOW ALL WORK. NO WORK = NO CREDIT.

Consider the function: $g(x) = \ln(x)$.

(a) **(8 points)** Find the degree 3 Taylor polynomial of $g(x)$ centered at the point $a = 1$.

(b) **(4 points)** Use your answer to Part (a) to find an approximate value for ln(1.5). Include at least **8 decimal places** in your answer.

$\ln(1.5) \approx$ ______________________________

Note: We are not looking for the value of ln(1.5) that your calculator gives. If you write that figure down as your answer you will receive zero credit.

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Consider the function: $g(x) = \ln(x)$.

(c) **(8 points)** The Taylor Remainder Theorem states that if $P_N(x)$ is the degree N Taylor polynomial (centered at $x = a$) approximation to the function $g(x)$, then:

$$\left|g(b) - P_N(b)\right| \le \frac{M}{(N+1)!} \cdot \left|b - a\right|^{N+1},$$

where M is an appropriate constant. Use the Taylor Remainder Theorem to find a reasonable upper bound for the error that occurs when $P_3(x)$ and $a = 1$ are used to approximate the value of ln(1.5). Be sure to clearly indicate your final answer and show all of your work.

If you express your final answer as a decimal, include at least **8 decimal places**.

FINAL ANSWER:

ERROR $\le$ ____

2. 20 Points. SHOW ALL WORK. NO PARTIAL CREDIT WITHOUT WORK.

Consider the function: $h(x) = \dfrac{x}{9 - x^2}$.

(a) **(8 points)** Write down the Taylor series of the function $h(x)$ about $a = 0$. Express your final answer using Σ notation.

FINAL ANSWER:

$$\sum$$

Continued on the next page.

Consider the function: $h(x) = \dfrac{x}{9 - x^2}$.

(b) **(12 points)** Find the **interval** of convergence for the Taylor series of $h(x)$ about $a = 0$. Show your work and clearly indicate your final answer. No work = no credit.

FINAL ANSWER:

INTERVAL: ____

3. 30 Points. SHOW YOUR WORK. NO WORK = NO CREDIT.

Determine the convergence or divergence of each of the following series. If you do not justify your answer, you will get zero credit, even if you circle the correct final answer.

In each case, demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test*. Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work. As the final part of your answer in each part, CIRCLE either CONVERGES or DIVERGES.

(a) (10 points) $$\sum_{n=1}^{\infty} \frac{3}{n \cdot (n+3)}$$

JUSTIFICATION:

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

Demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test*. Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work. As the final part of your answer in each part, CIRCLE either CONVERGES or DIVERGES.

(b) **(10 points)** $$\sum_{n=1}^{\infty} \frac{\pi^n}{3^{2n} \cdot (2n)!}$$

JUSTIFICATION:

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

Demonstrate that your answer is correct in a step-by-step fashion using *an appropriate convergence test*. Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work. As the final part of your answer in each part, CIRCLE either CONVERGES or DIVERGES.

(c) **(10 points)** $\sum_{n=2}^{\infty} \frac{4}{n \cdot \sqrt{\ln(n)}}$. You may use the following fact: $\lim_{n \to \infty} \sqrt{\ln(|n|)} = +\infty$.

JUSTIFICATION:

FINAL ANSWER (CIRCLE ONE): CONVERGES DIVERGES

4. 20 Points. SHOW YOUR WORK.

Consider the alternating series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}.$$

(a) **(10 points)** Does the series converge absolutely, converge conditionally or diverge? Clearly state your answer and use a convergence test to demonstrate that your answer is correct.

Be sure to explicitly state which convergence test you have used and show that it can be used with the series you are working on. Be careful to show all of your work and clearly state your final conclusion.

Continued on the next page.

The alternating series from the previous page is:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}.$$

(b) **(6 points)** Let S represent the sum of the series. Suppose that S, the sum of the series, is approximated by the N^{th} partial sum:

$$S_N = \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n^5}.$$

What is the smallest value of N that could be used to approximate S by S_N and an error of less than 0.0001?

(c) **(4 points)** Approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ with an error of less than 0.0001. Include at least **8 decimal places** in your answer.

5. 10 Points. SHOW ALL WORK AND CLEARLY INDICATE THE FINAL ANSWER.

Consider the infinite series:

$$\sum_{n=1}^{\infty} \left(\ln(x)\right)^n .$$

For what values of x does this series converge?

FINAL ANSWER:

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