SOLUTIONS

Math 120 **Winter 2009**

Quiz #8

1. In this problem, the function $f(x)$ will always refer to the function defined by the equation:

$$f(x) = e^{-x^2}.$$

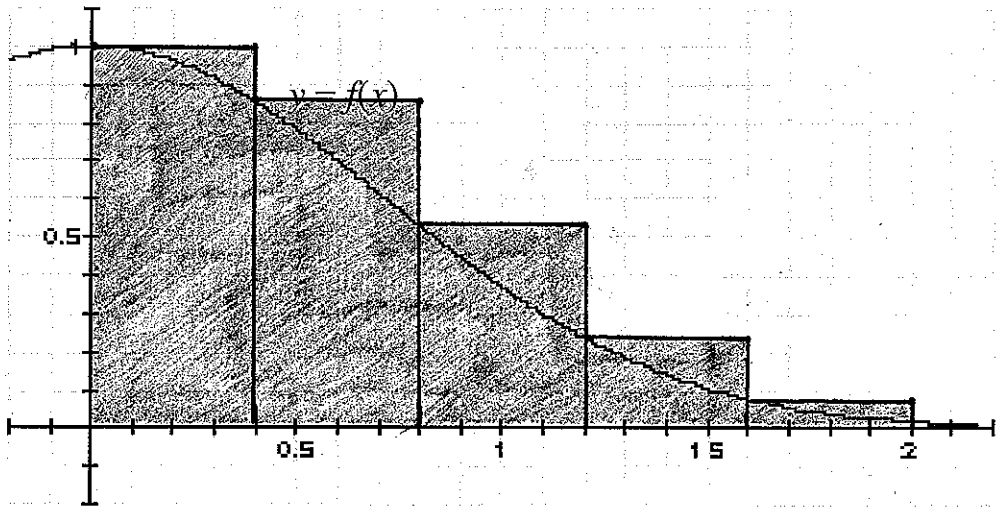
(a) **(1 point)** Use derivatives to determine whether or not the function: $F(x) = \dfrac{e^{-x^2+1}}{-x^2+1} + C$ is an anti-derivative of $f(x)$ or not. Be sure to show all of your work.

If $F(x)$ is an antiderivative of $f(x)$ then $F'(x) = f(x)$. Testing this:

$$F'(x) = \frac{(-x^2+1)\cdot e^{-x^2+1}\cdot(-2x) - (-2x)e^{-x^2+1}}{(-x^2+1)^2}$$

$$= \frac{-x^2\cdot e^{-x^2+1}}{(-x^2+1)^2} \neq e^{-x^2}.$$

(b) **(2 points)** The graph shown below is a graph of $y = f(x)$. Suppose that you were asked to approximate the area under the graph of $y = f(x)$ between $x = 0$ and $x = 2$ using five (5) rectangles. Use the graph provided below to sketch the five rectangles that you would use.



I have drawn a left hand sum here. You could also draw a right hand sum.

Continued on the next page

SOLUTIONS

(c) **(2 points)** Table 1 (below) lists the values of $f(x)$ at several different x-values. Use the entries in Table 1 to approximate the area beneath $y = f(x)$ between $x = 0$ and $x = 2$ using five (5) rectangles.

x	$f(x)$	x	$f(x)$
0	1	1.2	0.23693
0.2	0.96079	1.4	0.14086
0.4	0.85214	1.6	0.0773
0.6	0.69768	1.8	0.03916
0.8	0.52729	2.0	0.01832
1.0	0.36788		

Table 1: Selected values of the function $f(x)$.

You may not need to use all of the numbers from Table 1. Use only those that you actually need. Clearly indicate your final answer and include at least four (4) decimal places with your answer.

$\Delta x = \frac{2-0}{5} = 0.4.$

For the left hand sum drawn:

Area $\approx 0.4(1 + 0.85214 + 0.52729 + 0.23693 + 0.0773)$

$= 1.077464.$

For the right hand sum, Area $\approx$

$0.4(0.85214 + 0.52729 + 0.23693 + 0.0773 + 0.01832) = 0.684792$

(d) **(1 point)** Is the estimate that you calculated in Part (d) an under-estimate or an over-estimate of the "true value" of the area beneath $y = f(x)$ from $x = 0$ to $x = 2$? In a sentence or two, briefly explain how you know.

The left hand sum is an over-estimate. This is because $f(x)$ is a decreasing function between $x = 0$ and $x = 2$.

The right hand sum will give an under-estimate for the same reason.

SOLUTIONS

2. **(4 points)** Use **Newton's method** to find the solution of the equation:

$$\cos(x) = x.$$

correct to eight (8) decimal places. In order to get credit, you must show your work.

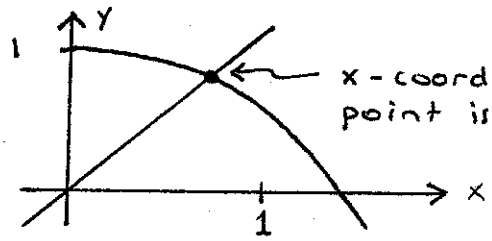
You should not use your calculator on this problem for anything except simple arithmetic, and evaluating trigonometric functions. In particular, you should not solve the equation by tracing the graph of a function or funding intersection points on your calculator.

If you use your calculator, make sure it is set in RADIAN mode. Clearly indicate your final answer and include at least eight (8) decimal places.

Write $f(x) = \cos(x) - x$.

Then $f'(x) = -\sin(x) - 1$

To guess x_1, sketch $y = \cos(x)$ and $y = x$:



x-coordinate of intersection point is close to $x = 1$. Use $x_1 = 1$.

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
1	1	-0.45969769	-1.841470985	0.7503638678
2	0.7503638678	-0.018923073	-1.68190495 3	0.7391128909
3	0.7391128909	$-4.6455899 \times 10^{-5}$	-1.673632544	0.7390851334
4	0.7390851334	-2.8471×10^{-10}	-1.673612029	0.7390851332

As x_5 and x_4 share the first 8 decimal places, we can stop here and give the final answer:

$$x \approx 0.73908513 32.$$

Additional space for your work is provided on the next page.