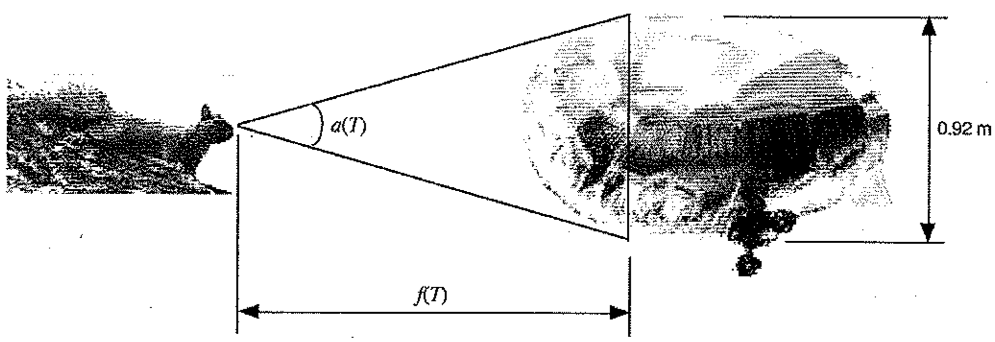
SOLUTIONS

Math 120 **Winter 2009**

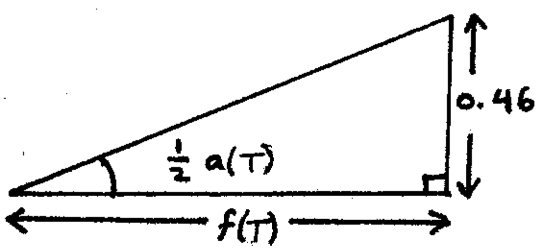
Quiz #6

1. Many creatures exhibit a behavior called an *escape response.* When frightened, the creature will accelerate very suddenly and dart away to avoid some perceived threat. A classic example of an animal escape response is exhibited by many species of squid, which use their powerful siphons to suddenly "jet" away from threats. As an additional form of protection, squid often release a cloud of camouflaging ink as they escape. One of the visual cues[1] thought to stimulate an escape response is the rate of change of the *apparent angle* of an approaching object.

The diagram[2] given below shows an object approaching a squirrel (*Sciurus carolinensis*). The object is moving and the squirrel is sitting still. The distance between the squirrel and the object is represented by the function $f(T)$. The apparent angle of the object (as seen by the squirrel) is represented by the function $a(T)$. T represents the time (in seconds) since the squirrel first spotted the approaching object.



(a) **(2 points)** The diameter of the object is 0.92 meters. Find an equation for the apparent angle, $a(T)$. Your equation may include the function $f(T)$.



Using trigonometry:

$$\frac{0.46}{f(T)} = \tan\left(\frac{1}{2}a(T)\right)$$

Rearranging to make a(t) the subject of the formula:

$$a(T) = 2 \cdot \tan^{-1}\left(\frac{0.46}{f(T)}\right).$$

[1] For example, see: Lee, D.N. 1976. A theory of visual control of braking based on information about time-to-collision. *Perception*, 5: 437-459.
For a perspective from the neurosciences, see: Sun, H. and B.J. Frost. 1998. Manipulation of different optical variables of looming objects in pigeon nucleus rotundus neurons. *Nature Neuroscience*, 1(4): 296-303.
[2] This figure was created using images from: http://www.hammacher.com/

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(b) **(2 points)** The object travels with a speed[3] of 0.38 meters per second. How quickly is the apparent angle changing when the object is 3 meters from the squirrel? (Your answer should be given in units of radians per second.)

$$a'(T) = 2 \cdot \frac{1}{1 + \left(\frac{0.46}{f(T)}\right)^2} \cdot \frac{-0.46}{f(T)^2} \cdot f'(T)$$

Have: $f'(T) = -0.38$ m/s.

$f(T) = 3$ m.

Plugging these into the formula for $a'(T)$ gives:

$a'(T) \approx 0.0379$ radians / second.

[3] Thanks to Professor Albert Chau (Department of Mathematics, Harvard University) for assistance with this measurement.

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2. In each case, determine whether the limit exists. If the limit exists, find its value.

(a) **(2 points)** $\lim_{x \to 0} \frac{\tan(x)}{\ln(1+x)}$.

$\tan(0) = 0$

$\ln(1+0) = \ln(1) = 0.$

$\frac{0}{0}$ indeterminant form so we can use L'Hôpital's rule.

$$\lim_{x \to 0} \frac{\tan(x)}{\ln(1+x)} = \lim_{x \to 0} \frac{\sec^2(x)}{\frac{1}{1+x}} = 1.$$

(b) **(2 points)** $\lim_{x \to 0} \frac{1-e^{-2x}}{\sec(x)}$.

$1 - e^{-2(0)} = 0$

$\sec(0) = 1.$

This is not an indeterminant form.

$$\lim_{x \to 0} \frac{1 - e^{-2x}}{\sec(x)} = \frac{1-1}{1} = 0.$$

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In each case, determine whether the limit exists. If the limit exists, find its value.

(c) (2 points) $\lim_{x \to 0^+} x^2 \cdot \ln(x)$

$$\lim_{x \to 0^+} \ln(x) = -\infty \qquad \lim_{x \to 0^+} \frac{1}{x^2} = +\infty.$$

If we rewrite the limit as:

$$\lim_{x \to 0^+} x^2 \cdot \ln(x) = \lim_{x \to 0^+} \frac{\ln(x)}{\frac{1}{x^2}}$$

which is an $\frac{\infty}{\infty}$ indeterminant form so we can use L'Hôpital's rule.

$$\lim_{x \to 0^+} x^2 \cdot \ln(x) = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$= \lim_{x \to 0^+} \frac{1}{x} \cdot \frac{x^3}{-2}$$

$$= \lim_{x \to 0^+} \frac{x^2}{-2}$$

$$= 0.$$