SOLUTIONS

Quiz #5

1. Use the technique of Implicit Differentiation to find an expression for $\frac{dy}{dx}$ in each case. Clearly indicate your final answers.

(a) **(3 points)** $x \cdot y + y^2 = \ln(x)$

$$y + x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = \frac{1}{x}$$

$$(x + 2y) \cdot \frac{dy}{dx} = \frac{1}{x} - y$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} - y}{x + 2y}$$

SOLUTIONS

Use the technique of Implicit Differentiation to find an expression for $\frac{dy}{dx}$ in each case. Clearly indicate your final answers.

(b) **(3 points)** $x \cdot \sqrt{y} + y \cdot \sqrt{x} = 1$

$$\sqrt{y} + x \cdot \tfrac{1}{2} y^{-1/2} \cdot \frac{dy}{dx} + \frac{dy}{dx} \cdot \sqrt{x} + y \cdot \tfrac{1}{2} x^{-1/2} = 0$$

$$\frac{x}{2\sqrt{y}} \frac{dy}{dx} + \sqrt{x} \frac{dy}{dx} = -\sqrt{y} + \frac{y}{2\sqrt{x}}$$

$$\left(\frac{x}{2\sqrt{y}} + \sqrt{x} \right) \frac{dy}{dx} = -\sqrt{y} + \frac{y}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y} + \frac{y}{2\sqrt{x}}}{\frac{x}{2\sqrt{y}} + \sqrt{x}}$$

2. **(4 points)** During filming of the Emmy award-winning documentary *Carrier: Fortress At Sea*, an F-14 pilot flew directly over an aircraft carrier at the speed of sound, approximately 330 meters per second. The pilot was flying at an altitude of 500 meters above the deck of the carrier. Because it is unusual for an aircraft to fly so fast near the carrier, many members of the crew gathered on the deck to watch[1] the fly-by. Figure 5[2] shows the *sighting angle, α,* of a crew member observing the fly-by. The person watching the fly-by is about two meters tall.

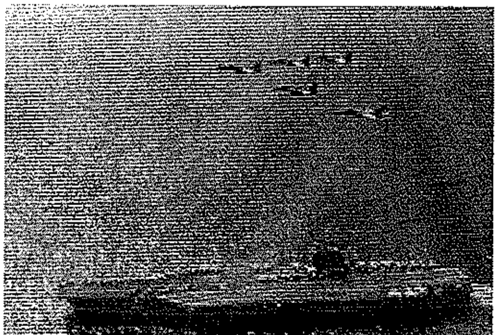


Figure 4: A group of F-14 "tomcat" fighters fly directly over a U.S. Navy aircraft carrier.

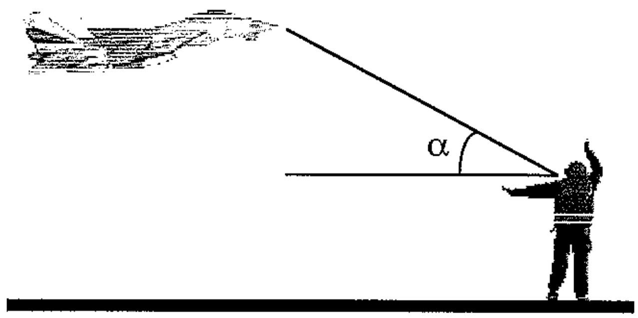
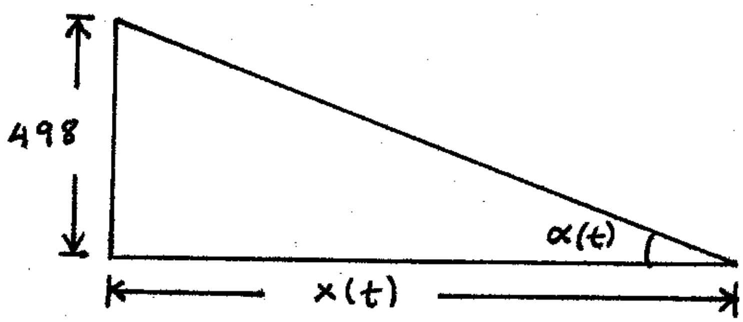


Figure 5: Diagram showing the sighting angle, α, as the crew member watches the F-14 fly overhead.

Calculate the rate at which the angle α is changing as the F-14 passes directly over the crewmember's head.



Want $\alpha'(t)$ when $x(t) = 0$.

$$\tan(\alpha(t)) = \frac{498}{x(t)}$$

$$\frac{1}{\cos^2(\alpha(t))} \cdot \alpha'(t) = \frac{-498}{x(t)^2} \cdot x'(t)$$

So: $$\alpha'(t) = \frac{-498 \cdot \cos^2(\alpha(t))}{x(t)^2} \cdot x'(t).$$

Now, $x'(t) = -330$ m/s and $\cos^2(\alpha(t)) = \dfrac{x(t)^2}{498^2 + x(t)^2}$

so: $$\alpha'(t) = \frac{-498}{498^2 + x(t)^2} \cdot (-330), \quad x(t) \neq 0.$$

$$\lim_{x(t) \to 0} \frac{(-498)(-330)}{498^2 + x(t)^2} = 0.6627 \text{ radians/second.}$$

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[1] And listen. At the speed of sound an aircraft produces an impressively loud noise called a "sonic boom" as it passes.

[2] This diagram was created using images from http://www.chinfo.navy.mil/