SOLUTIONS

Quiz #4

1. Calculate each of the following limits. Show your work or supply evidence for your answers – no work or evidence = no credit.

(a) **(1.5 points)** $\displaystyle \lim_{x \to \infty} \frac{x^5+2}{x^4+x^2+1}$

$$\lim_{x\to\infty} \frac{\frac{1}{x^4}(x^5+2)}{\frac{1}{x^4}(x^4+x^2+1)} = \lim_{x\to\infty} \frac{x + \frac{2}{x^4}}{1+\frac{1}{x^2}+\frac{1}{x^4}}$$

$$= \lim_{x\to\infty} x$$

$$= +\infty$$

(b) **(1.5 points)** $\displaystyle \lim_{x \to \infty} \frac{\sin^2(x)}{x^2+10}$

Note that for every value of x, $0 \leq \sin^2(x) \leq 1$.

Then, dividing by x^2+10 gives: $\displaystyle 0 \leq \frac{\sin^2(x)}{x^2+10} \leq \frac{1}{x^2+10}$.

Now, $\displaystyle \lim_{x\to\infty} 0 = 0$ and $\displaystyle \lim_{x\to\infty} \frac{1}{x^2+10} = 0$.

The squeeze lemma then gives $\displaystyle \lim_{x\to\infty} \frac{\sin^2(x)}{x^2+10} = 0$.

SOLUTIONS

(c) (1.5 points) $\lim\limits_{z \to -\infty} \dfrac{5z^4+20}{(z^2-4)\cdot(3z^2+1)}$

Foiling the denominator gives:

$$(z^2-4)\cdot(3z^2+1) = 3z^4 - 11z^2 - 4.$$

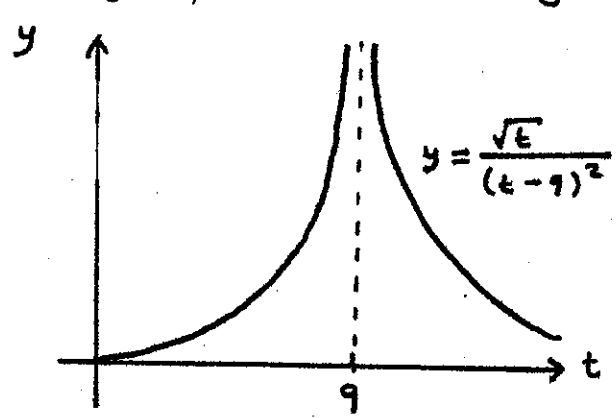
Therefore:

$$\lim_{z \to -\infty} \frac{\frac{1}{z^4}(5z^4+20)}{\frac{1}{z^4}(3z^4-11z^2-4)} = \lim_{z \to -\infty} \frac{5 + \frac{20}{z^4}}{3 - \frac{11}{z^2} - \frac{4}{z^4}}$$

$$= \frac{5}{3}$$

(d) (1.5 points) $\lim\limits_{t \to 9} \dfrac{\sqrt{t}}{(t-9)^2}$

If we graph $y = \dfrac{\sqrt{t}}{(t-9)^2}$ near $t = 9$, we get a graph resembling:



As t approaches 9 from either left or right, the graph grows to higher and higher y-values without bound.

$$\lim_{t \to 9} \frac{\sqrt{t}}{(t-9)^2} = \infty.$$

SOLUTIONS

2. In this problem you will always be concerned with the function: $f(x) = x^2 + 1$.

(a) **(2 points)** Use the definition of the derivative (i.e. no short-cut derivative rules):

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

to find the value of $f'(3)$, i.e. the derivative of $f(x)$ when $a = 3$. Show your work – no work = no credit.

$$f(3+h) = (3+h)^2 + 1 = 9 + 6h + h^2 + 1$$
$$= 10 + 6h + h^2$$

$$f(3) = 3^2 + 1 = 10$$

$$\frac{f(3+h) - f(3)}{h} = \frac{10 + 6h + h^2 - 10}{h}$$
$$= \frac{6h + h^2}{h}$$
$$= 6 + h, \quad \text{provided } h \neq 0.$$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$
$$= \lim_{h \to 0} 6 + h$$
$$= 6.$$

SOLUTIONS

(b) **(1 point)** Find an equation for the tangent line that touches the curve $y = f(x)$ at $x = 3$.

$$y - 10 = 6 \cdot (x - 3)$$

(c) **(1 point)** Use the diagram provided below to sketch an accurate graph of the line that you found in Part (b). The curve shown is $y = f(x)$.