Unit Test 3 Review Problems – Set B

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. Sketch a graph of a single function, $f(x)$, that has all of the following features:

- When $x < 2$, $f'(x) > 0$.
- When $2 < x < 4$, $f'(x) < 0$.
- When $x > 4$, $f'(x) > 0$.
- When $x < 3$, $f''(x) < 0$.
- When $3 < x < 5$, $f''(x) > 0$.
- When $x > 5$, $f''(x) < 0$.
- $f(0) = 4$.

NOTE: You do *not* need to come up with an equation for $f(x)$, all you need to do is produce a graph of $y = f(x)$ that shows all of the features listed above.

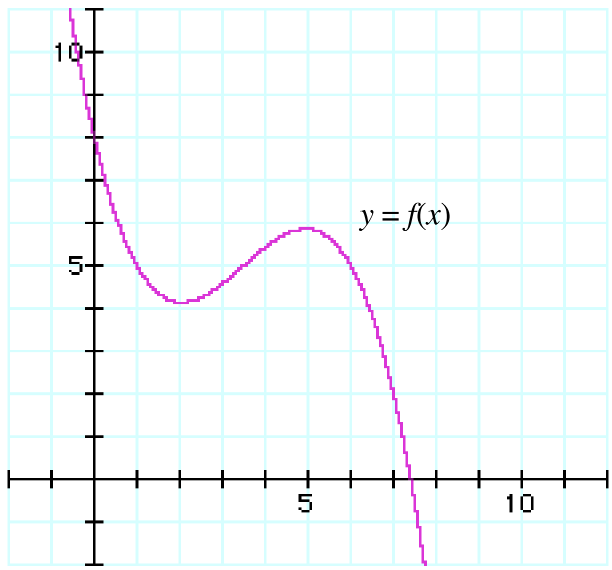
2. “Yankee Home Furnishings” is a small, family-owned furniture company that specializes in handcrafted furniture in a traditional “Shaker” style. Most of their business is with individual customers who want particular items of custom furniture made to their individual specifications. Occasionally, the company manufactures larger orders for corporate customers. In a major deal last year, the company agreed to deliver at least 300 and a maximum of 400 chairs for a major hotel chain. (The hotel chain was not sure of the exact number of chairs that they needed, and the deal was written so that they would get at least 300 chairs and more if they needed them.) The base price was \$90 per chair up to 300 chairs. If the hotel chain wanted more than 300 chairs, then the price would be reduced by 25 cents per chair (on the whole order) for every additional chair over 300 that the hotel chain ordered. What is the largest and the smallest revenue that Yankee Home Furnishings could make from this deal?

3. In this problem, the function $f(x)$ will always refer to the function defined by the graph shown on the next page.

(a) In terms of area under the curve and the ways that you have learned to approximate such areas, what does the symbolic statement:

$$\sum_{k=0}^{3} f(k \cdot 2) \cdot 2$$

represent?



(b) Sketch the area that is actually represented by the symbolic statement $\sum_{k=0}^{3} f(k \cdot 2) \cdot 2$.

(c) Using the graph given above to find the values of $f(x)$ that you need, find the numerical value of the symbolic statement: $\sum_{k=0}^{3} f(k \cdot 2) \cdot 2$.

(d) The equation of the function $f(x)$ is:

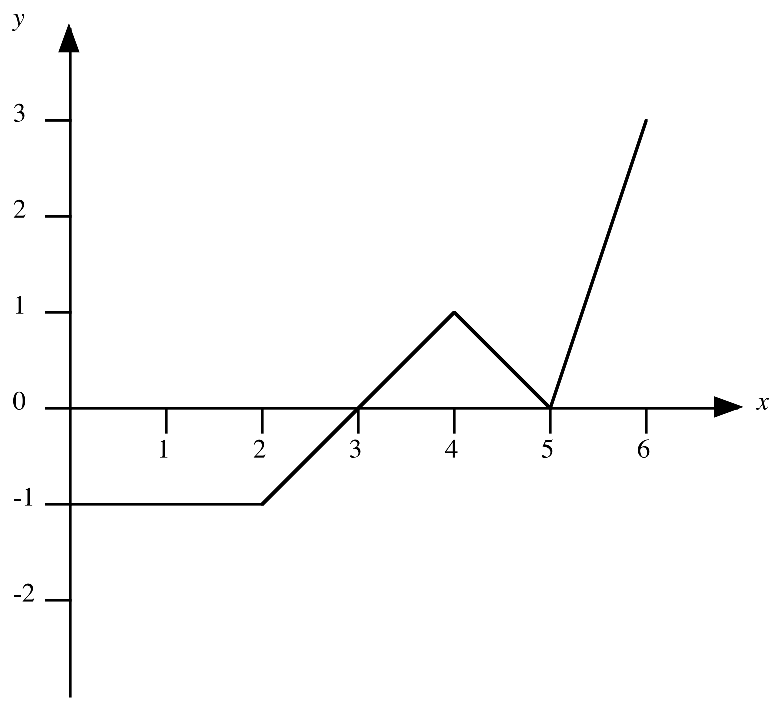
$$y = \tfrac{-1}{7} \cdot (x-1) \cdot (x-3.5) \cdot (x-6) + 5.$$

Use the "sum" and "seq" commands on your calculator to approximate the area beneath $y = f(x)$ between $x = 0$ and $x = 8$.

(e) Using integral notation, write down a symbolic statement that will represent the precise "true" value of the area between the curve $y = f(x)$ and the x-axis from $x = 0$ to $x = 8$.

(f) Use anti-derivatives to find the numerical value of the symbolic statement that you wrote down in Part (e) of this question.

4. The graph shown on the next page is the graph of $y = f(x) = F'(x)$.



(a) Locate and classify the critical points of $F(x)$.

(b) Locate the x-coordinates of the global maximum and minimum of $F(x)$ on the interval [0, 6].

(c) Locate any inflection points of $F(x)$. How do you know where the inflection points of $F(x)$ will occur?

(d) Suppose that the one other thing that you are told is that $F(2) = 14$. Sketch a graph of $y = F(x)$. Label the points that you have found in parts (a)-(d) of this problem.

5. In each of the following examples, decide whether $F(x)$ is an anti-derivative of $f(x)$ or not. If $F(x)$ is an anti-derivative of $f(x)$, then use $F(x)$ to evaluate:

$$\int_2^{10} f(x) \cdot dx.$$

(a) $F(x) = 2$ and $f(x) = x^2$.

(b) $F(x) = x \cdot \ln(x) - x$ and $f(x) = \ln(x)$.

(c) $F(x) = \frac{1}{1+x^2}$ and $f(x) = {-2x}/{(1+x^2)^2}$.

(d) $F(x) = \ln(\ln(x))$ and $f(x) = \dfrac{1}{\ln(x)}$.

(e) $F(x) = \dfrac{1}{\ln(x)}$ and $f(x) = \dfrac{-1}{x \cdot [\ln(x)]^2}$.

6. The definite integral, $\int_0^{\pi^2/4} \sin(\sqrt{x})dx$ cannot be calculated by taking an antiderivative, because the function being integrated does not have a straight-forward antiderivative. The value of the definite integral can be approximated using Riemann sums.

(a) You can calculate the number of rectangles needed to achieve a desired level of accuracy with a left-hand or right-hand Riemann sum. To approximate the definite integral $\int_a^b f(x)dx$ with an error of less than *E*, you should use *N* rectangles, where *N* satisfies the equation:

$$\frac{|f(b) - f(a)|}{N} \cdot (b-a) \leq E.$$

How many rectangles should be used so that the value of the definite integral given above can be estimated with an error of at most 0.15?

(b) Find the *left* and *right* Riemann sums, using the number of rectangles that you calculated in Part (a).

(c) Find a "best estimate" for the value of $\int_0^{\pi^2/4} \sin(\sqrt{x})dx$.

(d) Find the *midpoint* Riemann sum using the number of rectangles calculated in Part (a). Verify that the *midpoint* Riemann sum is not the average of the left and right Riemann sums.

7. The function $y = f(x)$ is defined by the following differential equation and initial condition:

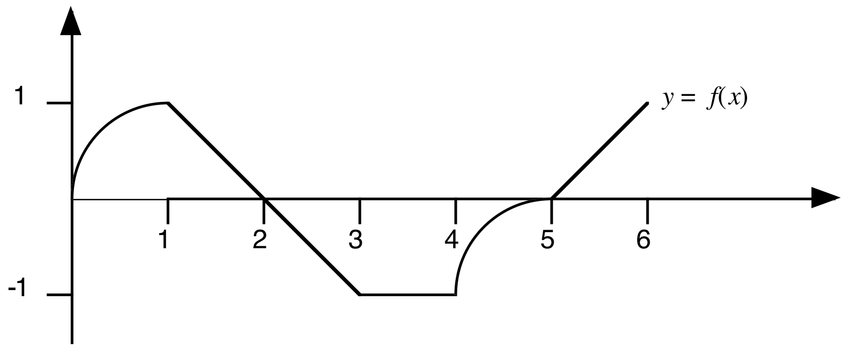
Differential equation: $f'(x) = x^2 + [f(x)]^2$ or $y' = x^2 + y^2$

Initial value: $f(0) = 1$ or $y(0) = 1$.

(a) Use Euler's Method and $\Delta x = 0.25$ to estimate $f(1)$.

(b) Is the estimate of $f(1)$ that you calculated in Question (a) an over-estimate or an under-estimate of the actual value of $f(1)$? Be careful to explain how you know.

8. A function $F(x)$ has the properties that $F(5) = 6$, and $F'(x) = f(x)$. The function $f(x)$ is graphed below. Find the value of $F(x)$ when $x = 1, 2, 3$, and 4.



9. Use the method of u-substitution to find formulas for each of the following indefinite integrals. Remember that finding a formula for an indefinite integral is the same as finding the most general anti-derivative (so don't forget to add the "+*C*" to the end of each of your formulas!).

(a) $\int \frac{2^x}{7+2^x} \cdot dx.$

(b) $\int (4x-6) \cdot (x^2 - 3x + 8)^2 \cdot dx.$

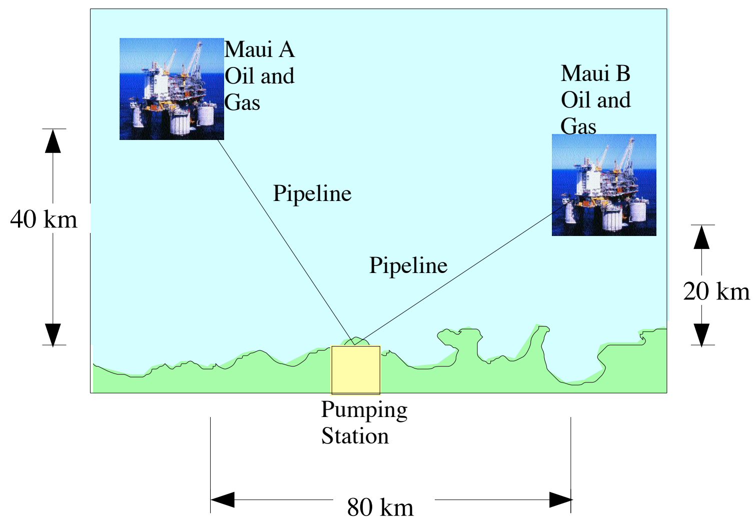
(c) $\int \frac{x}{(1+x^2)^2} \cdot dx.$

(d) $\int \frac{9x^2}{\sqrt{1-x^3}} \cdot dx.$

(e) $\int \frac{14x}{7x^2+7} \cdot dx.$

(f) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \cdot dx.$

10. The diagram given below shows part of the province of Taranaki on the west coast of the North Island of New Zealand. Taranaki has been nicknamed the "Energy Province" of New Zealand because reserves of petroleum and natural gas have been located there. The two main natural gas fields are called "Maui A" and "Maui B." Spurred by the "oil shocks" of the late 1970's the New Zealand government began to set up the pipelines and stations needed to extract natural gas and oil from the Maui fields. The most popular plan was to build two pipelines that ran to a central pumping station on shore.



The Maui A platform was located 40 km off the coast and the Maui B platform was 20 km off the coast. The distance between the two platforms (along the coastline) was 80 km. Where was the most cost-effective place to build the pumping station?