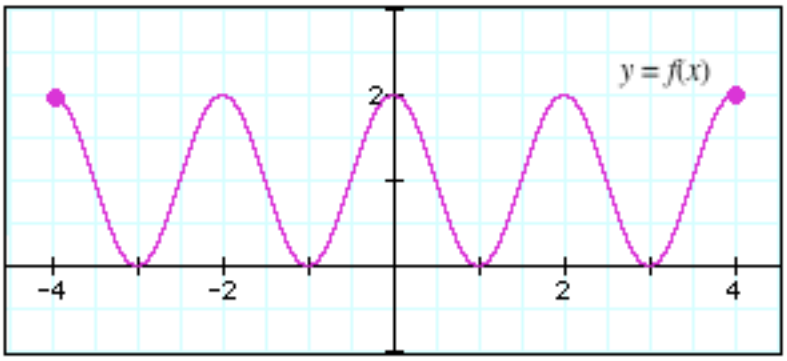
Unit Test 3 Review Problems – Set A

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. In this problem, the function $f(x)$ will always refer to the function defined by the graph given below. The domain of the function $f(x)$ is the interval $[-4, 4]$. Note that this interval does include the end-points $x = -4$ and $x = 4$.



In this problem, the function $g(x)$ will always refer to the function defined by the equation:

$$g(x) = [f(x)]^2 - 2 \cdot f(x).$$

(a) What is the domain of the function $g(x)$?

(b) Find the x-coordinates of the point(s) where the derivative of $f(x)$ is equal to zero.

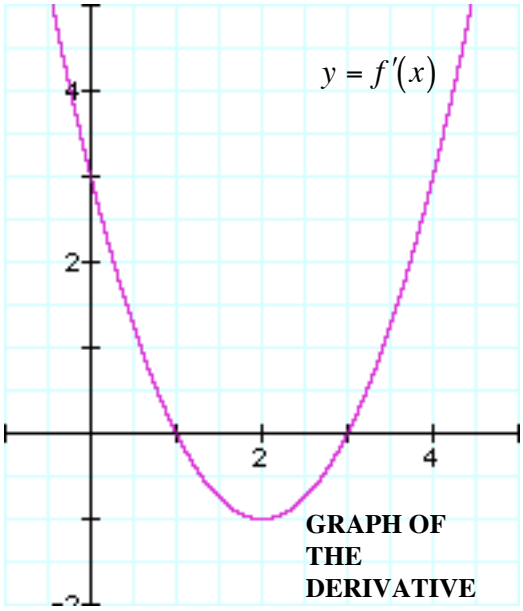
(c) Express the derivative of $g(x)$, $g'(x)$, in terms of $f(x)$ and the derivative of $f(x)$.

(d) Find the x-coordinates of all points where the derivative of $g(x)$ is equal to zero.

(e) For each of the points that you found in Part (d), determine whether the point is a local maximum, local minimum or neither.

2. After an unsuccessful campaign to market oil in long, rolled-up tubes (called "Coil 'O' Oil"), a company resorts to selling their oil in more conventional, cylindrical containers. The company plans to sell 500 ml cylindrical cans of oil, but the packaging material for the ends of the can costs 2 cents per square centimeter, whereas the material for the sides costs only one cent per square centimeter. If the company only cares about how much the packaging costs, what should the dimensions of their oil can be?

3. The graph given below is the graph of the **derivative** of a function $f(x)$. In this problem the function $f(x)$ will always refer to a function that has a derivative identical to the one shown in the graph below.



(a) Find the x-coordinates of all of the critical points of the function $f(x)$.

(b) Classify each of the critical points that you found as a minimum or a maximum. In each case, be careful to explain what information you are using from the derivative graph to make your decision.

(c) Find the x-coordinates of any points where the **concavity** of the original function $f(x)$ changes. Briefly explain how you know that the concavity changes at the point(s) you have located.

(d) Suppose the one other thing that you know about the function $f(x)$ is that $f(0) = 1$. Sketch a possible graph of $y = f(x)$ that is consistent with the derivative graph. Be careful to indicate the locations of all of the points that you found in Parts (a) and (c).

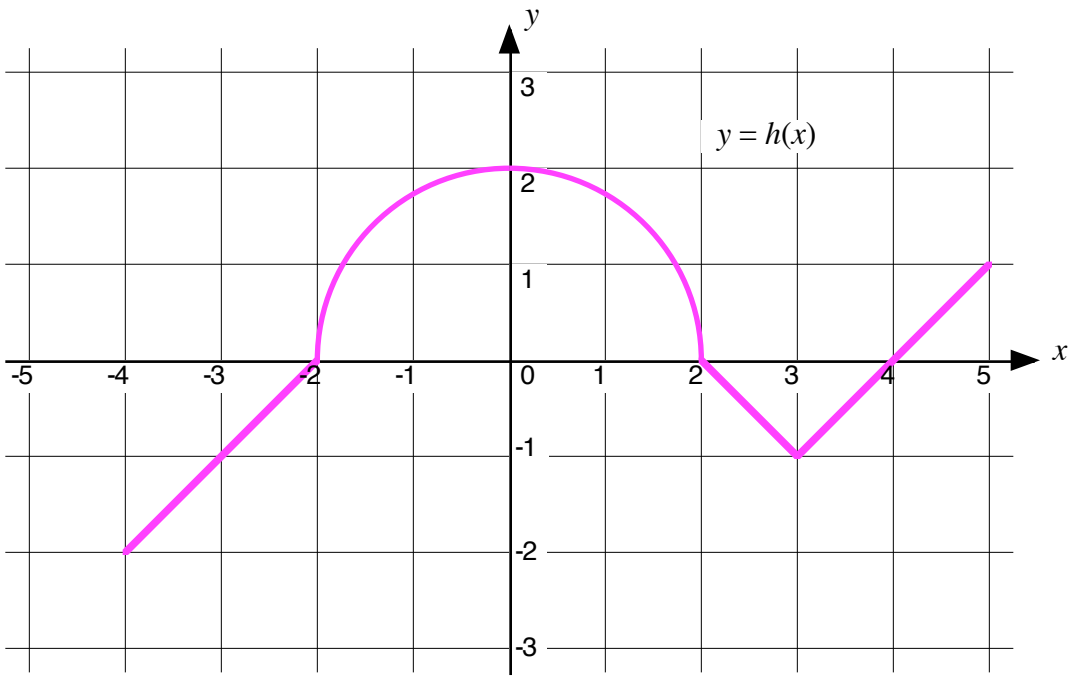
4. In this problem, the function $h(x)$ will always refer to the function defined by the graph shown on the next page. The graph of $y = h(x)$ is entirely composed of straight line segments and a semicircle.

(a) Shade the area on the graph that corresponds to the expression: $\int_0^5 h(x) \cdot dx$.

(b) Shade the area on the graph that corresponds to the expression: $\int_{-2}^{-1} h^{-1}(y) \cdot dy$.

(c) Calculate the exact numerical value of the symbolic expression: $\int_{0}^{2} h(x) \cdot dx$.

(d) Calculate the exact numerical value of the symbolic expression: $\int_{3}^{5} h(x) \cdot dx$.



5. In this problem the functions $f(x)$ and $F(x)$ will always refer to the functions defined by the formulas:

- $f(x) = x \cdot e^{-x}$
- $F(x) = -x \cdot e^{-x} - e^{-x}$.

In this problem, $g(x)$ is also a function. All that you can assume about the function $g(x)$ is listed below.

- $g(2) = 6$
- $g(5) = -3$
- $g'(2) = 4$
- $g'(5) = 1$

(a) Verify that $F(x)$ is an anti-derivative of $f(x)$. Be sure to show full details of your calculations.

(b) Evaluate the numerical value of the definite integral: $\int_{0}^{2} f(x) \cdot dx$.

(c) A new function $h(x)$ is defined by the equation:

$$h(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x).$$

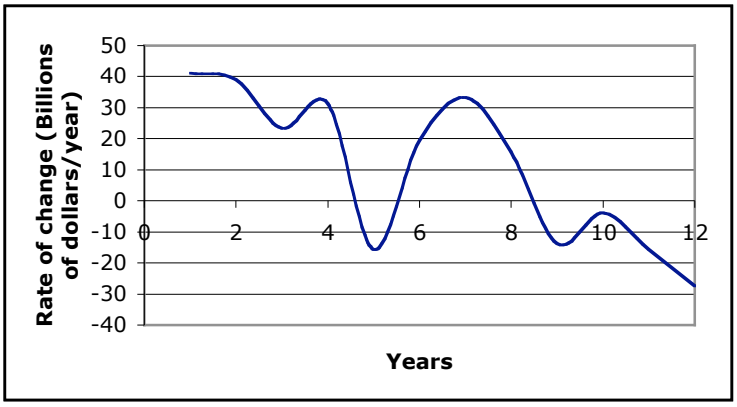
Find a formula for the most general anti-derivative of $h(x)$. Your answer should contain one unspecified constant (i.e. $+C$).

(d) Evaluate the exact numerical value of the definite integral: $\int_2^5 h(x) \cdot dx$.

6. In 1913, Argentina was one of the world's ten richest countries (and was wealthier than either Germany or France). However, during the 20th century Argentina's standing as a world power slipped considerably. In 1991 Argentine President Carlos Menem and economy minister Domingo Cavallo set out to reverse this decline.

The **gross domestic product** (GDP) of a country is a measure of the total economic output of a country. The **rate of change** of Argentina's GDP[1] is shown in the graph given below.

In this problem let T represent the number of years since 1990 and $r(T)$ the **rate of change** of Argentina's GDP in units[2] of billions of dollars per year.



Graph showing **RATE OF CHANGE** for GDP in Argentina, 1991-2002.

(a) The units of the vertical axis of the graph are "billions of dollars per year." The units of the horizontal axis of the graph are "years." What are the units of the area under the curve?

(b) What does the area under the curve between $T = 1$ and $T = 12$ represent (in terms of Argentina, years, billions of dollars and GDP)?

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[1] Source: "A decline without parallel." *The Economist*, March 2 2002, pp 26-28.

[2] In case you were wondering, the "dollars" referred to here are 2002 U.S. dollars. If this doesn't mean anything to you, don't worry – this fact has no relevance to your calculation.

(c) According to the graph, what has been happening to the Argentine GDP in the last four years (i.e. T = 9, 10, 11 and 12)? In a sentence or two, briefly explain how you know.

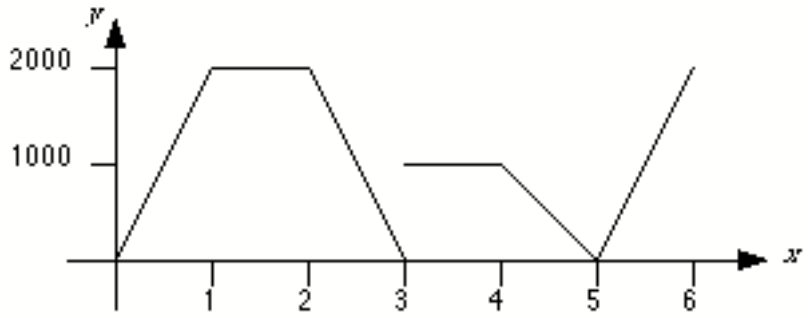
(d) The curve shown in the graph can be approximated by the cubic equation:

$$r(T) = -0.2 \cdot T^3 + 2.5 \cdot T^2 - 17.7 \cdot T + 59.1.$$

Find an equation for the anti-derivative, $R(T)$, of this cubic equation. Your anti-derivative should contain one unspecified constant (i.e. "+C").

(e) Calculate the total change in the Argentine GDP between 1991 and 2002. Show details of your work and make sure that you include units with your answer.

7. Suppose that $F(0) = 2$ and that $F'(x) = f(x)$. The graph of $y = f(x)$ is given below. The lemming is a small rodent found in northern climes.



(a) Calculate the value of $F(x)$ for x = 1, 2, 3, 4 and 5.

(b) Suppose that x has units of days, and $f(x)$ has the units of lemmings per day. What are the units of $F(x)$?

(c) In terms of lemmings, explain the possible meaning of x, $f(x)$ and $F(x)$ in practical terms.

8. In this problem, you will approximate the integral: $\int_0^3 e^{\sqrt{x}} \cdot \sin(x) \cdot dx$ using a variety of different methods. For Parts (a)-(e) of this problem, calculate the approximate value of the integral using the method named and the number of rectangles specified. It is fine to use your calculator to evaluate the sums that you get.

NOTE: Make sure your calculator is in RADIAN mode before you start these calculations.

(a) Left hand Riemann sum, 100 rectangles.

(b) Right hand Riemann sum, 200 rectangles.

(c) Midpoint sum, 300 rectangles.

(d) Trapezoid rule, 400 rectangles. (This is the average of the left-hand and right-hand sums with 400 rectangles each.)

9. A company is testing the idea of producing and marketing a new video game console over the Internet. The same company will both manufacture and sell the game consoles. Each video game console retails for \$256, while the cost of production of the game consoles (in dollars) is given by the function:

$$C(q) = 1000 + q^3,$$

where q is the number of game consoles manufactured and sold.

(a) Write down the profit function for the video game consoles.

(b) Assuming that all consoles produced will be sold, how many consoles should the company produce if they are mostly interested in maximizing their profit?

(c) Suppose that production is now fixed at 9 video game consoles per week (and the costs fixed at the cost of producing 9 consoles). Market forces dictate that at a price of \$256, exactly 9 toys will be sold each week. The company's advisers claim that for every \$50 price rise, the number of consoles sold will go down by 2. What price should the company sell the video consoles for in order to maximize weekly profits?

10. In this problem you are required to calculate formulas for each of the anti-derivatives (or indefinite integrals) listed below. Each of these problems can be solved using the technique of u-substitution, although you are welcome to use whatever methods you choose. Your answers should each include an unspecified constant along the lines of "$+C$."

(a) $\int 20 \cdot \left(x^2 + x + 1\right)^{19} \cdot \left[2x + 1\right] \cdot dx$

(b) $\int \frac{1}{2} \cdot \left(x + \ln(x)\right)^{\frac{-1}{2}} \cdot \left[1 + \frac{1}{x}\right] \cdot dx$

(c) $\int \frac{6x^2 + 4}{x^3 + 2x + 10} \cdot dx$

(d) $\int \frac{e^{\sqrt{x}}}{2 \cdot \sqrt{x}} \cdot dx$