Math 120, Winter 2009. Answers to Unit Test 2 Review Problems – Set B.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) The best graph is the bottom, left-hand graph.

1.(b) The range is $1 \le p \le 3$.

1.(c) $g(350)$ is the price of a Luther burger when 350 burgers are sold at a baseball game.

1.(d) $g^{-1}(2.5)$ is the number of Luther burgers that are sold at a baseball game when the price of a burger is \$2.50.

1.(e) $q = 450/p - 50$.

1.(f) The domain of $g^{-1}(p)$ is the same as the range of q.

2.(a) $S(2) = 0.8485$ cubic meters. When two inches of rain have fallen, 0.8485 cubic meters of soil have been eroded.

2.(b) $R^{-1}(3.4) = 1.5$ minutes. 90 seconds after the start of the thunderstorm, 3.4 inches of rain have fallen.

2.(c) $S(R(1.5)) = S(3.4) = 1.8807$ cubic meters. 90 seconds after the start of the thunderstorm, 1.8807 cubic meters of soil have been eroded.

2.(d) $R^{-1}(S^{-1}(2.4)) = 2$ minutes. Two minutes after the start of the thunderstorm, 2.4 cubic meters of soil have been eroded.

3.(a) $dy/dx = (y - xy\ln(y))/(x^2 + 3xy^3)$.

3.(b) $dy/dx = 0$.

3.(c) $dy/dx = (y^2 + x^4y^4 - 2xy)/(x^2 - 2xy - 2x^5y^3)$.

3.(d) $dy/dx = (a - x)/y$.

4.(a) The annual percentage change is 3.5634%.

4.(b) It will take 6.14 years.

4.(c) It will happen when $t = 2.9$ years. This would be the end of 2011 or beginning of 2012 depending on how you round off and which day you count as $t = 0$.

5.(a) Approximately 3.8685 hours.

5.(b) The relic is probably about 686 years old. It is unlikely that it is more than 2000 years old.

5.(c) After an additional 32.36 (32 or 33) days.

5.(d) Approximately 2.40942 minutes.

6.(a) The derivative is $10x^4 + 9x^2 + 1$.

6.(b) As the value of the derivative is always greater than zero, the function $f(x)$ is always increasing. This means that $f(x)$ will pass the horizontal line test.

6.(c) 6.

6.(d) 20.

6.(e) 0.05.

7.(a) The limit is equal to zero.

7.(b) The limit is equal to zero.

7.(c) The limit is equal to zero.

7.(d) The limit is equal to zero.

8.(a) After approximately 6.07 minutes.

8.(b) The person probably died at about 10:50pm.

9.(a) $h(t) = 300 - 30t$.

9.(b) $\theta(t) = \tan^{-1}((200 - 30t)/150)$. $\dfrac{d\theta}{dt} = \dfrac{-1}{5}\left(\dfrac{150^2}{150^2 + \left(200 - 30t\right)^2}\right)$.

9.(c) The elevator appears to be moving fastest when it is 100 feet from the ground (this occurs when $t = 20/3$ seconds).

10.(a) The highest points are those located at $x = \pm T/w$. At these points $y = (T/w)\cosh(1)$. The lowest point is at $x = 0$. At this point $y = T/w$. Subtracting these gives the sag as:

Sag = $(T/w)(\cosh(1) - 1)$.

(b) Using the Chain Rule, the first derivative of y with respect to x is equal to:

$$\frac{dy}{dx} = \sinh\left(\frac{wx}{T}\right).$$

Using the Chain Rule a second time, the second derivative of y with respect to x is equal to:

$$\frac{d^2y}{dx^2} = \frac{w}{T}\cosh\left(\frac{wx}{T}\right).$$

Now, noting that $\cosh^2(x) = \sinh^2(x) + 1$, the other side of the given equation simplifies to:

$$\frac{w}{T}\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \frac{w}{T}\sqrt{1+\sinh^2\left(\frac{wx}{T}\right)} = \frac{w}{T}\sqrt{\cosh^2\left(\frac{wx}{T}\right)} = \frac{w}{T}\cosh\left(\frac{wx}{T}\right).$$

This is equal to the second derivative given above.