Unit Test 2 Review Problems – Set B

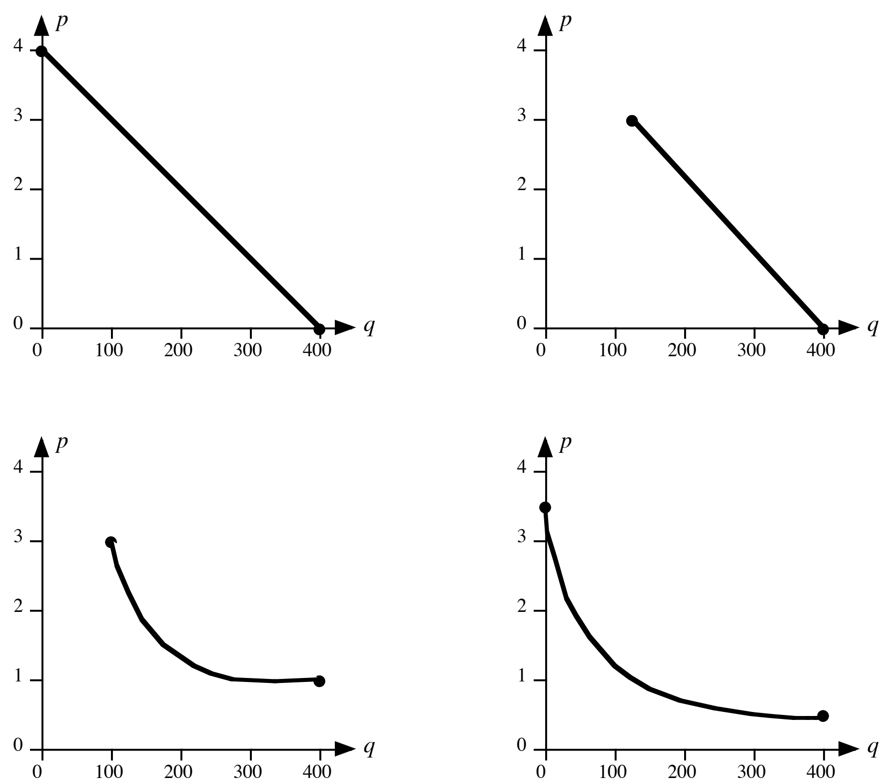
We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. A "Luther burger" is a bacon cheeseburger in which the hamburger bun is replaced by a Krispy Kreme donut. Luther burgers are served at the home games of an Illinois minor league baseball team. Let p be the price of a Luther burger (in dollars) and q the number of Luther burgers sold during a baseball game. Then p and q are related by the function:

$$p = g(q) = \frac{9}{1 + 0.02q}$$

Domain: $100 \le q \le 400$.

(a) Which of the following diagrams does the best job of showing the relationship between p and q?



(b) Find the range of $g(q)$ that will make practical sense in the context of this problem.

(c) Explain the meaning of $g(350)$ in practical terms, such as dollars and Luther burgers.

(d) Explain the meaning of $g^{-1}(2.5)$ in practical terms, such as dollars and Luther burgers.

(e) Find a formula for $q = g^{-1}(p)$. Show all work.

(f) Find the domain of $g^{-1}(p)$ that will make practical sense in the context of this problem.

2. The cubic meters S of soil eroded from a certain hillside is a function of the number of inches of rain D that have fallen during a thunderstorm. It is given by the formula

$$S(D) = 0.3D^{\frac{3}{2}}.$$

The rainfall R in inches t minutes after the thunderstorm has begun is given by the following table.

t (minutes)	0	.25	.5	1	1.5	2
R (inches)	0.0	1.1	2.0	2.4	3.4	4.0

(a) Calculate $S(2)$ and give its units. Explain its practical meaning in words.

(b) Calculate $R^{-1}(3.4)$ and give its units. Explain its practical meaning in words.

(c) Calculate $S(R(1.5))$ and give its units. Explain its practical meaning in words.

(d) Calculate $R^{-1}(S^{-1}(2.4))$ and give its units. Explain its practical meaning in words.

3. In each case, find a formula for dy/dx. You may assume that a, b and c are constants.

(a) $x\ln(y) + y^3 = \ln(x)$.

(b) $\cos^2(y) + \sin^2(y) = y + 2$.

(c) $\tan^{-1}(x^2y) = xy^2$.

(d) $(x - a)^2 + y^2 = b^2$.

4. In a Michigan town, house prices are falling. Over the last two years the house prices have fallen by 7%. You may assume that the annual percentage change in house prices is constant.

(a) What is the annual percentage change in house prices? State your answer as a percentage and give at least two decimal places of accuracy.

(b) How long will it take for the price of a house in this town to decrease by 20%? Show your work. Give your answer to at least two decimal places of accuracy.

(c) Suppose that a house is currently (in 2009) worth $200,000. In what year will the house be worth only $180,000? Show your work. Give your answer to at least two decimal places of accuracy.

5. Each of the following scenarios involves exponential growth or decay. Use exponential functions and logarithms to answer the question(s) posed in each scenario.

(a) In a certain culture of bacteria, the number of bacteria increases six fold in 10 hours. How long did it take the bacteria to double their number?

(b) The half-life of carbon-14 is 5730 years. A modern object contains 5.0×10^{10} carbon-14 atoms per gram. An ancient relic contains 4.6×10^{10} carbon-14 atoms per gram. If an antiques dealer claimed that the relic was more than 2000 years old, could this be true?

(c) Generalized Motors has discontinued advertising of their range of SUVs. The company plans to resume advertising when sales have declined to 75% of their initial rate. After one week without advertising, sales declined to 95% of their initial rate. When should the company resume advertising?

(d) When sugar is dissolved in water, the amount A of sugar that remains undissolved after t minutes satisfies the differential equation $dA/dt = -kA$ where $k > 0$ is a positive constant. If 25% of the sugar dissolves in one minute, how long does it take for half the sugar to dissolve?

6. In this problem, $f(x) = 2x^5 + 3x^3 + x$.

(a) Find a formula for the derivative $f'(x)$.

(b) How can you use your answer to Part (a) to decide whether the inverse of $f(x)$ will be a function or not?

(c) Find $f(1)$.

(d) Find $f'(1)$.

(e) Find $\left(f^{-1}\right)'(6)$.

7. For each of the following limits, determine whether or not the limit exists, and if so, find its value.

(a) $\displaystyle \lim_{t \to \pi} \frac{\sin^2(t)}{t-\pi}$.

(b) $\displaystyle \lim_{x \to 0^+} x \cdot \ln(x)$.

(c) $\displaystyle \lim_{u \to 0} \frac{1-\cosh(3u)}{u}$.

(d) $\displaystyle \lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$.

8. Suppose that the temperature of an environment is a constant, *E*, and the temperature of an object that is hotter or cooler than the environment is given by a function *T*(*t*), where *t* is the amount of time that has passed. The rate of change of the object's temperature is described by a differential equation called Newton's Law of Cooling:

$$T'(t) = -k \cdot [T(t) - E],$$

where *k* is a positive constant. The solution of this differential equation looks like:

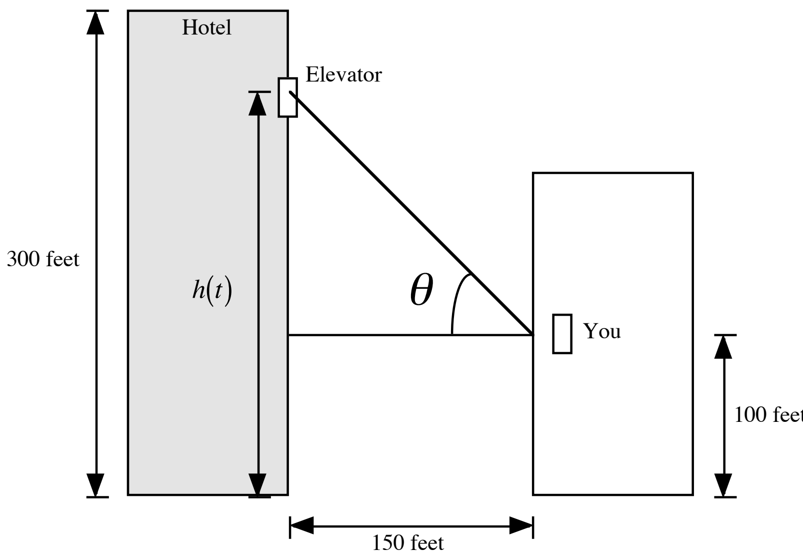
$$T(t) = E + A \cdot e^{-kt},$$

where *A* is a constant. Use this information to answer the questions given below.

(a) Suppose a cup of coffee has an initial temperature of 200°F when freshly poured. The cup is in a room that has a temperature of 70°F. One minute after it was poured, the coffee reaches a temperature of 190°F. How long does it take for the cup of coffee to reach 150°F?

(b) A dead body is discovered at midnight. When it is discovered, an investigator determines that the temperature of the body is 85°F. The body was in a room that had a temperature of 68°F and it remains in that room as the investigation into the cause of death takes place. Two hours later, the investigator takes the temperature of the body again and finds that it is now (at 2am) 74°F. When, approximately, did the person die?

9. For the entertainment of its guests, a hotel has a glass elevator that goes up and down the outside of the building (see diagram given below). The hotel is 300 feet tall.



You are watching the elevator from a room in a building located 150 feet from the hotel. Your room is 100 feet from the ground. At time $t = 0$, the elevator is at the top of the hotel and immediately begins to descend at a constant speed of 30 feet per second. Let θ represent the angle between your horizon and your line of sight to the elevator.

(a) Find a formula for $h(t)$, the elevator's height above the ground as it descends from the top of the hotel.

(b) Use your answer from Part (a) to express θ as a function of time t and find the rate of change of θ with respect to time.

(c) The rate of change of θ with respect to time is a measure of how fast the elevator appears to be moving as you watch it. At what height above the ground is the elevator when it appears to be moving the most quickly to you?

10. The cable between two towers of a power line hangs in the shape of the curve:

$$y = \frac{T}{w}\cosh\left(\frac{wx}{T}\right),$$

where T is the tension in the cable at its lowest point and w is the weight of the cable per foot of length. (T and w are constants.)

(a) Suppose that the cable stretches between the points $x = -T/w$ and $x = T/w$. Find an expression for the "sag" in the cable. (That is, find the difference between the height of the cable at the highest and lowest points.)

(b) Show that the shape of the cable satisfies the equation:

$$\frac{d^2y}{dx^2} = \frac{w}{T}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$