Unit Test 2 Review Problems – Set A

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. In each case, find a formula for dy/dx. You may assume that a, b and c are constants.

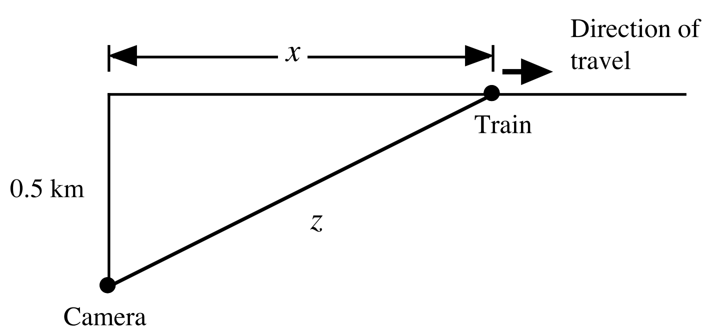
(a) $xy + x + y = 5$.

(b) $\sqrt{x} = 5\sqrt{y}$.

(c) $xy - x - 3y - 4 = 0$.

(d) $ax^2 - by^2 = c^2$.

2. A train is traveling at 0.8 km/minute along a long, straight track, moving in the direction shown in the diagram below. A movie camera, 0.5 km away from the track, is focused on the train.



(a) Express z, the distance between the camera and the train, as a function of x.

(b) How far is the distance from the camera to the train changing when the train is 1 km from the camera? Include units with your answer.

(c) How fast is the camera rotating (in radians per minute) at the moment when the train is 1km from the camera?

3. The Great Barrier Reef is a large coral reef in Australia. In 1996, the area of healthy coral on the reef was 135,000 square miles. In 2006, Australian scientists estimated that 45% of the healthy coral present in 1996 had died.

(a) How many square miles of coral died between 1996 and 2006?

(b) How many square miles of healthy coral existed on the Great Barrier Reef in 2006?

(c) Write an exponential formula approximating N, the number of square miles of healthy coral t years after 1996.

(d) What was the annual percentage decay rate between 1996 and 2006?

(e) In what year will there be only 10,000 square miles of healthy coral on the Great Barrier Reef? Round your answer to the nearest year.

4. Find the equation of the tangent line for each of the curves listed below at the point indicated.

(a) $x^2 + y^2 = 1$ at (0, 1).

(b) $x^3 + 2xy + y^2 = 4$ at (1, 1).

(c) $xy^2 = 1$ at (1, –1).

(d) $y^2 = x^2/(xy - 4)$ at (4, 2).

5. Solve each of the following equations for x. You should use algebra and logarithms (where appropriate) to solve the equations. You should not use your calculator to solve these equations except for performing arithmetic and working out the numerical values of logarithms, square roots, etc. You can use either common or natural logarithms.

(a) $2e^{\frac{\sqrt{x}}{2}} = 3e^{\frac{\sqrt{x}}{3}}$

(b) $\left(10^x\right)^2 = \sqrt{5}$

(c) $\ln(x-5) + \ln(5) = 0$

(d) $\frac{\log(x)+1}{\log(x)-1} = 3.$

6. The table given below gives some of the values of a function $f(x)$ and its derivatives. Use the values given in the table to answer the questions in this problem.

x	0	1	2	3
f(x)	1	2	4	8
f'(x)	0.7	1.4	2.8	5.5

(a) Calculate the values of the following quantities:

(I) $f(2)$ (II) $f'(2)$

(III) $f^{-1}(2)$ (IV) $\left(f^{-1}\right)'(2)$.

(b) Find the equation of the tangent line to $y = f(x)$ at the point (3, 8).

(c) Find the equation of the tangent line to $y = f^{1}(x)$ at the point (8, 3).

7. A train is heading due west from St. Louis. At noon, a plane flying horizontally due north at a fixed altitude of 4 miles passes directly over the train. When the train has traveled another mile, it is going 80 miles per hour, and the plane has traveled another 5 miles and is going 500 miles per hour. At that moment, how fast is the distance between the train and the plane increasing?

8. For each of the following limits, determine whether or not the limit exists, and if so, find its value.

(a) $\displaystyle \lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{e^t - 1} \right).$ **(b)** $\displaystyle \lim_{t \to 0} \frac{\sin^2(A \cdot t)}{\cos(A \cdot t) - 1}, A \neq 0.$

(c) $\displaystyle \lim_{x \to 0^+} x^a \cdot \ln(x), a > 0.$ **(d)** $\displaystyle \lim_{x \to 0} \frac{\cos(x)}{x}.$

9. A ladder 41 feet long that was leaning against a vertical wall begins to slip. Its top slides down the wall while its bottom moves along the level ground at a constant speed of 4 feet per second. How fast is the top of the ladder moving when it is 9 feet above the ground?

10. The length of each side of a cube is increased at a constant rate. Which is greater, the relative rate of change of the volume of the cube:

$$\frac{1}{V}\frac{dV}{dt},$$

or the relative rate of change of the surface area of the cube:

$$\frac{1}{A}\frac{dA}{dt}?$$

Explain (with the help of some calculations) how you know.