Math 120, Winter 2009. Answers to Unit Test 1 Review Problems – Set B.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $P = 12.5n + 1100$.

1.(b) For every additional person who places an order (or for every additional order places), the profit increases by \$12.50.

1.(c) If not people place orders in a given week the weekly profit for the web site is \$1100.

1.(d) \$12.50.

2.(a) To calculate $g'(1)$ will substitute the given function and derivative values into the expression above. This gives the following.

$$g'(1) = \frac{f'(1)\cdot 1 - 1\cdot f(1)}{1^2} = 0$$

2.(b) To calculate $g'(2)$ will substitute the given function and derivative values into the expression above. This gives the following.

$$g'(2) = \frac{f'(2)\cdot 2 - 1\cdot f(2)}{2^2} = \frac{-1}{2}$$

2.(c) To calculate the derivative of $m(x)$ it is appropriate to use the product rule for derivatives. The Product rule gives the following formula for the derivative of $m(x)$.

$$m'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x).$$

Substituting the function and derivative values given above (including the value of $g'(2)$ from Part (b)) gives the following.

$$m'(2) = f'(2)\cdot g(2) + f(2)\cdot g'(2) = \frac{-3}{4}$$

2.(d) There are several ways to approach this calculation. One of the most direct is to recognize that the definition of $g(x)$ simplifies the formula for $k(x)$ quite a lot.

$$k(x) = \frac{g(x)}{f(x)} = \frac{1}{f(x)}\cdot g(x) = \frac{1}{f(x)}\cdot\frac{f(x)}{x} = \frac{1}{x}.$$

Using the Power rule for derivatives gives that $k'(x) = -1 \cdot x^{-2}$. Substituting x = 2 into this derivative formula gives that $k'(x) = \frac{-1}{4}$.

3.(a) Using the power rule for derivatives, the derivative is: $f'(x) = -1 \cdot x^{-2}$.

3.(b) Using the formula given for $f(x)$ to create the difference quotient gives the following.

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h}-\frac{1}{x}}{h}.$$

3.(c) When simplifying the difference quotient, your objective is always to reduce the difference quotient to the point where you are able to cancel out the h that appears in the denominator. This is because you will eventually have to take the limit as $h \to 0$, and having a factor of h left on the bottom of the fraction will make the limit calculation unnecessarily complicated.

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{x-(x+h)}{x\cdot(x+h)}}{h} = \frac{-h}{h\cdot x\cdot(x+h)}.$$

You can now cancel the h's to remove the factor of h in the denominator of the difference quotient.

$$\frac{f(x+h)-f(x)}{h} = \frac{-1}{x\cdot(x+h)}.$$

3.(d) The derivative is equal to the limit of the difference quotient as $h \to 0$. Taking the limit of the expression obtained in Part (c) of this problem gives the following.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{-1}{x\cdot(x+h)} = \frac{-1}{x^2}.$$

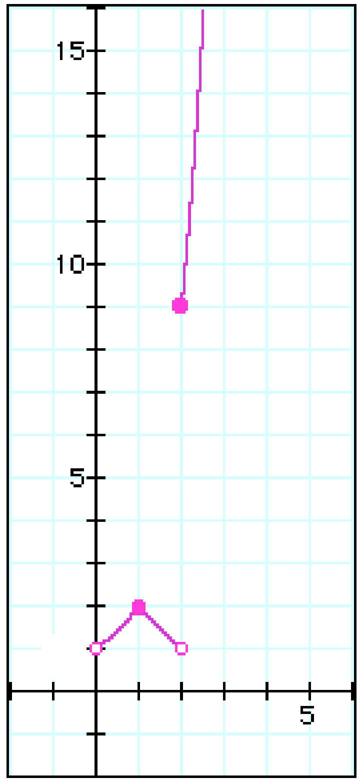
4.(a) $B(t-2)$.

4.(b) $B(t) + 2$.

4.(c) $B(2t)$

4.(d) $2B(t)$.

5.(a) The graph of $y = f(x)$ is shown below.



5.(b) The left and right hand limits of the function *f* are given in the table below.

x	Left hand limit of the function f	Right hand limit of the function f
0	Does not exist	1
1	2	2
2	1	9
3	27	Does not exist

Note that *f* does not have a left hand limit at $x = 0$ because *f* is not defined when $x \le 0$. Similarly, *f* does not have a right hand limit at $x = 3$ because *f* is not defined when $x > 3$.

5.(c) The answers that you get here will depend a lot on what you think it means for a *limit* to exist at a finite value of x. The official mathematical definitions are the following:

- If the x-value in question is the **left end point of an interval** then the *limit* exists provided that the right hand limit of the function exists there. The value of the *limit* is equal to the value of the right hand limit.

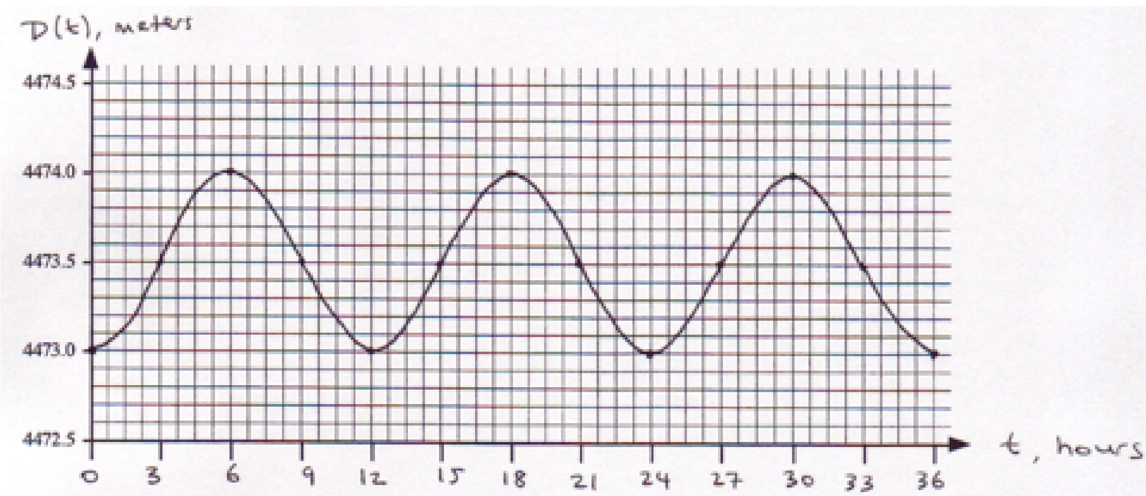
- If the x-value in question is the **right end point of an interval** then the *limit* exists provided that the left hand limit of the function exists there. The value of the *limit* is equal to the value of the left hand limit.

• If the x-value in question is not an end-point, then the *limit* exists at the x-value in question provided that the left hand limit and the right hand limit are equal.

Based on these criteria, the *limit* of f exists at every x-value from $x = 0$ to $x = 3$ (inclusive) with the exception of $x = 2$. The reason that the *limit* of f does not exist at $x = 2$ is that the left hand limit (= 1) and the right hand limit (= 9) are not equal.

Note: If you did not know about the convention for limits and end-points at intervals, you would also be justified in believing that the *limit* of f did not exist at either $x = 0$ or at $x = 3$ because at these two points one of the left hand or right hand limits does not exist.

6.(a) The graph of $D(t)$ is shown below.



6.(b) $D(t) = -0.5\cos\left(\frac{2\pi}{12}t\right) + 4473.5$

6.(c) $D(33) = 4473.5$ meters.

6.(d) $t = 33 + 24 = 57$. $D(57) = 4473.5$ meters. As 4474 exceeds this by more than 0.03 meters, a warning would have been issued.

7.(a) The limit is equal to 5.

7.(b) The limit does not exist as the height of the graph $y = f(x)$ does not settle down to any consistent value. Instead, it just oscillates back and forth.

7.(c) The limit does not exist as the left and right hand limits are not equal.

7.(d) The limit does not exist as the left and right hand limits are not equal.

7.(e) The limit is equal to 1.

7.(f) The limit does not exist as the left and right hand limits are not equal.

7.(g) The limit is equal to 1.

7.(h) The limit is equal to 2.

7.(i) The limit is equal to $+\infty$.

8.(a) $f'(x) = 2x$

8.(b) $f'(x) = \dfrac{-2}{(2x+1)^2}$

8.(c) $f'(x) = \dfrac{1}{\sqrt{2x+1}}$

8.(d) $f'(x) = \dfrac{1}{(1-2x)^2}$

9.(a) The limit is equal to 0.

9.(b) The limit is $+\infty$. (It would also be technically correct to say "Does not exist.")

9.(c) The limit is equal to -1.

9.(d) The limit is equal to 1.

9.(e) The limit is equal to 2.

9.(f) The limit is equal to -1.

10.(a) The key relationship here is the relationship between the area function, $A(t)$, and the radius function, $r(t)$:

$$A(t) = \pi \cdot r(t) \cdot r(t).$$

This is a *product* of two functions ($r(t)$ and $r(t)$) so when differentiating it is appropriate to use the *product rule*.

$$A'(t) = \pi \cdot r'(t) \cdot r(t) + \pi \cdot r(t) \cdot r'(t) = 2\pi \cdot r(t) \cdot r'(t).$$

This is the desired relationship between the derivative of the area function, $A'(t)$, the radius function, $r(t)$, and the derivative of the radius function, $r'(t)$.

10.(b) The quantity that you need to calculate here is: $r'(9)$. Rearranging the equation from Part (a) to make $r'(t)$ the subject gives the following:

$$r'(t) = \frac{A'(t)}{2 \cdot \pi \cdot r(t)}.$$

We were told that when $t = 9$, the radius had reached 50 cm. So $r(9) = 50$. From the graph of the instantaneous rate of change of the area, you can read off the value for the derivative at $t = 9$:

$$A'(t) = 38.$$

Substituting these values into the expression for $r'(t)$ gives:

$$r'(9) = \frac{A'(9)}{2 \cdot \pi \cdot r(9)} = \frac{38}{100 \cdot \pi} = 0.121 \text{ cm per hour.}$$

So at 9am, the radius of the cleared area beneath the acacia tree is increasing at a rate of approximately 0.121 cm per hour.