Unit Test 1 Review Problems – Set B

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1. A commercial web site makes money by selling products to people who place orders through the web site, and by charging companies to advertise on the web site. The manager of the web site graphed weekly profits as a function of the number of people who placed orders through web site and found that the relationship was linear. One week the profit was \$7600 when 520 people placed orders. Another week the profit was \$9975 when 710 people placed orders.

(a) Find a formula for P, the weekly profit in dollars, as a function of n, the number of people who place orders through the web site.

(b) Interpret the practical meaning of the slope of your function.

(c) Interpret the practical meaning of the P-intercept of your function.

(d) On average, how much profit does the web site make from each person who places an order through the web site?

2. In this problem the function $g(x)$ will always refer to the function defined by the equation:

$$g(x) = \frac{f(x)}{x}.$$

All that you can assume about the function $f(x)$ is that $f(x)$ has a derivative and that:

- $f(1) = 1$
- $f(2) = 3$
- $f'(1) = 1$
- $f'(2) = \frac{1}{2}$.

Use this information to calculate each of the derivatives described below.

(a) $g'(1)$

(b) $g'(2)$

(c) $m'(2)$ where $m(x) = f(x) \cdot g(x)$.

(d) $k'(2)$ where $k(x) = \dfrac{g(x)}{f(x)}$.

3. In this problem, the function $f(x)$ will always refer to the function defined by:

$$f(x) = \frac{1}{x}.$$

(a) Using any short-cuts or differentiation rules that you know, find an equation for the derivative $f'(x)$.

(b) Use the formula given for $f(x)$ to create and expression for the difference quotient,

$$\frac{f(x+h) - f(x)}{h}.$$

(c) Simplify the expression that you created in Part (b) until it is possible to cancel out the h in the denominator of the difference quotient.

(d) By simplifying the difference quotient as much as possible and taking the limit as $h \to 0$, find an equation for the derivative $f'(x)$.

4. After many years of hard work Belle fulfilled her dream of opening a bakery. Belle has an automated cookie-making machine that bakes all of the cookies that she sells. Belle's plan each day is to turn on the machine at 6am.

The function $B(t)$ represents the number of boxes of cookies that are ready to sell on a particular day t hours after 6am. Note that Belle does not run the machine all the time, and some types of cookies take longer to make than others, so $B(t)$ is not a linear function.

For each story, select the algebraic expression that does the best job of representing the number of boxes of cookies that are ready to sell. If you believe that none of the algebraic expressions correspond to a certain story, write **NONE** next to that story.

ALGEBRAIC EXPRESSIONS:

$B(t) + 2$	$B(t) - 2$	$B(t + 2)$	$B(t - 2)$
$2B(t)$	$0.5B(t)$	$B(2t)$	$B(0.5t)$

STORIES:

(a) One day, Belle is not feeling very well. She does not manage to turn on the cookie-making machine until 8am that day.

(b) One day Belle starts the cookie-making machine at 6am but has 2 boxes of cookies left over from the previous day to sell as well.

(c) When Belle's son comes home from college he helps out in the bakery. With her son helping to feed ingredients into the machine, the cookies are ready in half the usual time.

(d) Eventually, Belle is so successful that one machine is not enough. She has a second cookie-making machine installed and runs both of her machines in exactly the same way.

5. In this problem the function that you are interested in will always be the function $f(x)$ defined below.

$$f(t) = \begin{cases} 2^t & ,0 < t < 1 \\ 3 - t & ,1 \le t < 2 \\ 3^t & ,2 \le t \le 3 \end{cases}$$

(a) Sketch a graph of $y = f(x)$. Make sure that you label the end-points of each portion of your graph carefully.

(b) Find the right and left hand limits of the function f as x approaches the values listed below.

- $x = 0$
- $x = 1$
- $x = 2$
- $x = 3$

(c) Describe the set of x-values where the limit of the function f exists. For each point between $x = 0$ and $x = 3$ (inclusive) where the limit of f does not exist, briefly explain why the limit of f does not exist there.

6. The National Oceanic and Atmospheric Administration (NOAA) monitors sea depth with Deep-ocean Assessment and Reporting of Tsunamis (DART) buoys. A buoy recorded a minimum ocean depth of 4473 meters at 2:00 am on Oct 20. It records maximum ocean depth of 4474 meters at 8:00 am the same day, then records minimum ocean depth of 4473 meters at 2:00 pm on October 20, followed by maximum ocean depth of 4474 at 8:00pm of the same day. Under normal circumstances, this pattern repeats each day.

(a) Let $D(t)$ represent the depth of the ocean, in meters, measured by the buoy t hours after 2:00 am on October 20. Assuming that $D(t)$ is a sinusoidal function, sketch an accurate graph for $D(t)$ that shows ocean depth from 2:00 am on October 20 to 2:00 pm on October 21. Carefully label your axes and write a scale on the horizontal axis.

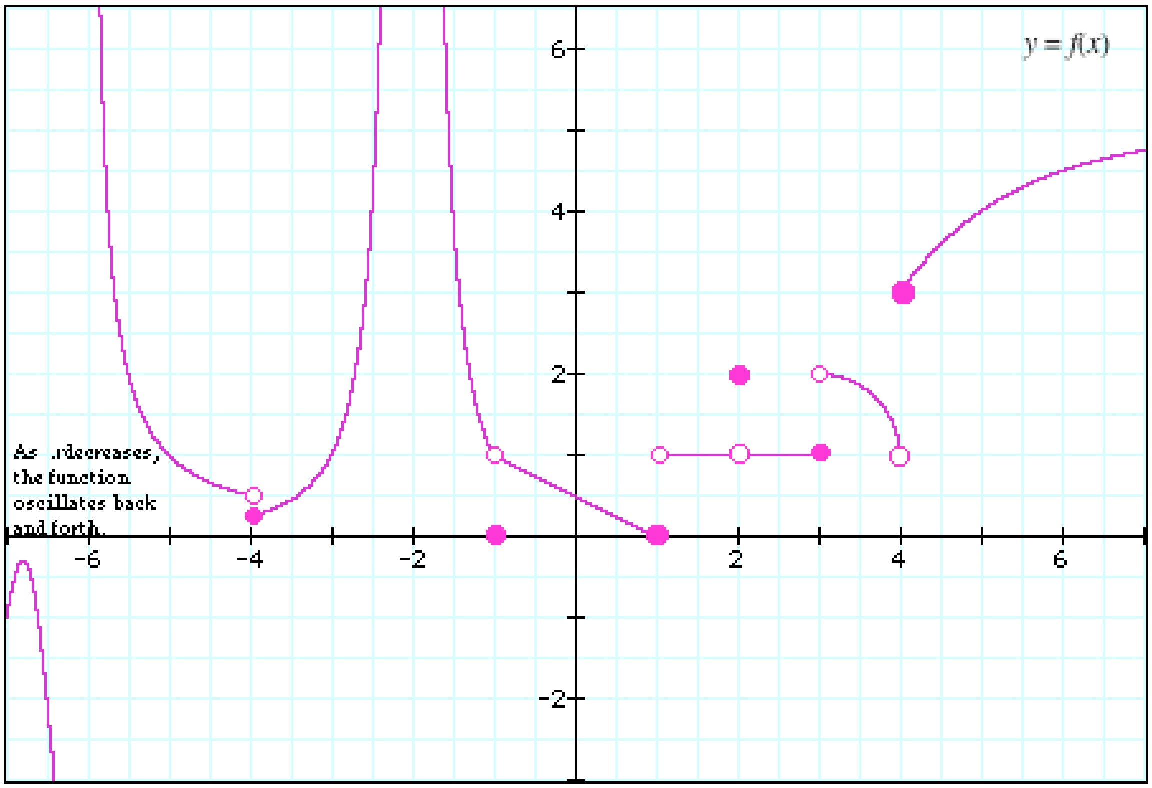
(b) Find a formula for $D(t)$.

(c) According to the formula in Part (b), what is the depth of the ocean at 11am on October 21?

Each DART buoy has a computer that compares the measured ocean depth to $D(t)$, the expected ocean depth. If the measured and expected values differ by more than 0.03 meters then the buoy will conclude that a tsunami has been detected and issue a warning.

(d) Suppose that the buoy measured a depth of 4474 meters at 11am on October 22. Will the buoy issue a warning at 11am on October 22? Support your conclusion with an appropriate calculation.

7. In this problem, the function *f* is the function defined by the graph shown below.



Evaluate the following limits. If the limit does not exist, explain why.

(a) $\lim_{x \to +\infty} f(x)$

(b) $\lim_{x \to -\infty} f(x)$

(c) $\lim_{x \to -4} f(x)$

(d) $\lim_{x \to -6} f(x)$

(e) $\lim_{x \to -1} f(x)$

(f) $\lim_{x \to 1} f(x)$

(g) $\lim_{x \to 2} f(x)$

(h) $\lim_{x \to 2} f(2)$

(i) $\lim_{x \to -2} f(x)$

8. Use the definition of the derivative to calculate the formula for $f'(x)$ for each of the following functions. You can use the short-cut rules to check your answer, but use the difference quotient and limit as $h \to 0$ to calculate the derivative.

(a) $f(x) = x^2 + 5$ **(b)** $f(x) = \dfrac{1}{2x+1}$

(c) $f(x) = \sqrt{2x+1}$ **(d)** $f(x) = \dfrac{x}{1-2x}$

9. Calculate each of the one-sided limits listed below.

(a) $\lim_{x \to 5^-} \sqrt{x(x-5)}$ **(b)** $\lim_{x \to 4^+} \sqrt{\dfrac{4x}{x-4}}$

(c) $\lim_{x \to 5^-} \dfrac{x-5}{|x-5|}$ **(d)** $\lim_{x \to 3^+} \dfrac{\sqrt{x^2-6x+9}}{x-3}$

(e) $\lim_{x \to 1^+} \dfrac{1-x^2}{1-x}$ **(f)** $\lim_{x \to 5^+} \dfrac{\sqrt{(5-x)^2}}{5-x}$

10. In the Costa Rican rainforest some species of ants (*Pseudomermex spinicola*, *Pseudomermex ferruginea* and *Pseudomermex nigrocinta* - see Figure 3[1]) live in acacia trees (*Acacia collinsi*). The tree provides the ants with shelter and the ants work to eliminate competition from other plants. One of the things that the ants do is to clear away plants from around the acacia tree, leaving a circular patch of bare earth around the tree (see Figure 4[2]). According to Janzen (1966) when the colony is sufficiently large (more than 1200 ants) ants clear the ground 24 hours per day.

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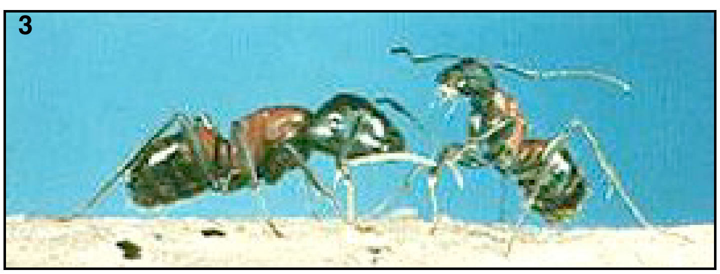

Figure 3: A pair of acacia ants (*Pseudomermex ferruginea*)

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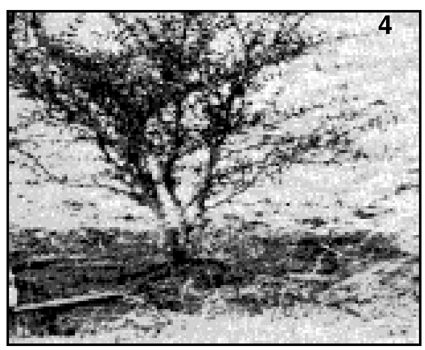

Figure 4: An acacia plant. The dark patch underneath the plant is the circle of bare earth that has been cleared by the ants.

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[1] Image source: http://www.abc.net.au/science/news/stories/s58491.htm

[2] Source: Janzen, D. H. (1966) "Coevolution of mutualism between ants and acacias in Central America." *Evolution*, **20**(3): 249-275.

Figure 5 (below) shows the ***INSTANTANEOUS RATE*** at which a large colony of ants clears the ground beneath an acacia tree. (The units of the instantaneous rate are square centimeters per hour.)

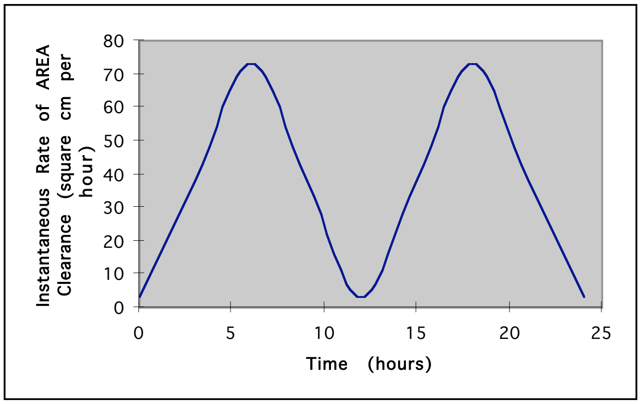

Figure 5: Graph showing instantaneous rate of change of AREA cleared.

(a) Suppose that $A(t)$ represents the amount of area (in square centimeters) cleared by the ants after they have been working for t hours. Let $r(t)$ represent the radius of the circle (in centimeters) that the ants have cleared after they have been working for t hours. Then $A(t)$ and $r(t)$ are related by the equation:

$$A(t) = \pi \cdot [r(t)]^2 = \pi \cdot r(t) \cdot r(t).$$

Find an equation that relates the derivative of the area function, $A'(t)$, to the radius function, $r(t)$, and the derivative of the radius function, $r'(t)$. Show details of your calculation.

(b) At 9am ($t = 9$) the radius of the circular area cleared by the ants had reached 50cm. What is the instantaneous rate of change of the *radius* of the circular area at 9am? Show details of your calculation.