Math 120, Winter 2009. Answers to Unit Test 1 Review Problems – Set A.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $y = \frac{1}{12}(x+3)(x+2)x^3(x-2)$

1.(b) $y = 3 \cdot \cos\left(\frac{2\pi}{10}x\right) + 6$

2.(a) 2

2.(b) 10

2.(c) 8

2.(d) 6

2.(e) 0

2.(f) 8

2.(g) $\dfrac{p(k(5)+h) - p(k(5))}{h} = \dfrac{8h + h^2}{h} = 8 + h$ with the last simplification valid when $h \neq 0$.

3.(a) $y = \frac{1}{2}(x-2)^2 + 1$

3.(b) The exact value of the x-intercept is $1/\cos(70^\circ)$, which is approximately 2.92.

4.(a) $f'(x) = \frac{3}{2}x^2 - 2x$.

4.(b) When $x = 0$, the graph of $y = g'(x)$ has height 3. This means that the derivative of g will be equal to three at that point. In symbols, that means that: $g'(0) = 3$. The derivative of $m(x)$ at $x = 0$ is: $m'(0) = f'(0) + g'(0) = 0 + 3 = 3$.

4.(c) $r(x) = m'(x) = f'(x) + g'(x) = \frac{3}{2}x^2 - 2x + 3$.

5.(a) $F(t) = \begin{cases} 450 & ,0 \le t \le 4 \\ 16t^2 - 240t + 1154 & ,4 < t \le 11 \\ 450 & ,11 < t < 15 \end{cases}$

5.(b) $u(t) = 270 - 15t$

5.(c) Katharine wins the election (505 votes to 450 votes).

6.(a) The completed table is given below.

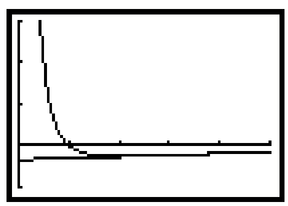
x	2.9	2.99	2.999	2.9999	3.0001	3.001	3.01	3.1
$\frac{f(x)-f(3)}{x-3}$	0.036 99	0.0370 37	0.03703 7	0.037037	0.037037	0.03703 7	0.0370 37	0.036 99

Based on the trend of the values in the table, the limiting value of $\frac{f(x)-f(3)}{x-3}$ as x approaches 3 appears to be equal to. Therefore the value of the derivative of $f(x)$ at x = 3 is approximately 0.037037.

6.(b) The slope of the tangent line is 0.037037, and the tangent line passes through the point (3, f(3)) = (3, -2/9). The equation of the line with slope 0.037037 that passes through the point (3, -2/9) is:

$$y = 0.037037x - 0.3333332222.$$

The graph of $y = f(x)$ and the graph of the tangent line is shown below.



6.(c) There are four steps that one normally needs to complete in order to calculate the derivative algebraically:

(a) Use the definition of the derivative to correctly find $f(a + h)$.

(b) Use the definition of the function to calculate the difference quotient, $\frac{f(a+h)-f(a)}{h}$.

(c) Simplify the difference quotient as much as possible. A good clue that you have done this is that there should be no factors of h remaining in the denominator of the difference quotient.

(d) Find the limiting value of the difference quotient as $h \to 0$.

Performing these steps, one after the other.

(a) Formulating $f(a+h)$:

$$f(x) = \frac{1-(a+h)}{(a+h)^2}.$$

(b) Calculating the difference quotient:

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{1-(a+h)}{(a+h)^2} - \frac{1-a}{a^2}}{h}.$$

(c) Simplifying the difference quotient gives:

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{1-(a+h)}{(a+h)^2} - \frac{1-a}{a^2}}{h} = \frac{\frac{a^2-a^2\cdot(a+h)-(a+h)^2+a\cdot(a+h)}{a^2\cdot(a+h)^2}}{h} = \frac{a^2h-2ah-h^2+ah^2}{ha^2(a+h)^2}.$$

and simplifying further gives:

$$\frac{f(a+h)-f(a)}{h} = \frac{a^2-2a-h+ah}{a^2(a+h)^2}.$$

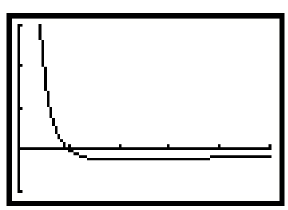
Note that the denominator of the difference quotient no longer contains any *factors* of h. There is an h still in the denominator, but it is not a factor by itself.

(d) Taking the limit as $h \to 0$ gives that the derivative of $f(x)$ at $x = a$ is equal to:

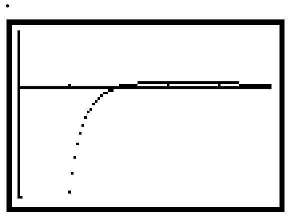
$$f'(a) = \frac{a^2-2a}{a^4} = \frac{1}{a^2} - \frac{2}{a^3}.$$

6.(d) The two graphs ($y = f(x)$ and $y = f'(x)$) are shown below

Graph of $y = f(x)$:



Graph of $y = f'(x)$:



7.(a) The limit is equal to −1/9.

7.(b) The limit is equal to 4.

7.(c) The limit is equal to ¼.

7.(d) The limit is equal to −32.

8.(a) $\lim_{x \to 1} f(x) = \frac{1}{2}$.

To deduce this limit, note that the denominator of f can be factored:

$$f(x) = \frac{1-x}{1-x^2} = \frac{1-x}{(1-x)\cdot(1+x)} = \frac{1-x}{1-x}\cdot\frac{1}{1+x}.$$

When x is close to 1, $f(x)$ will be very close to (1)·(1/2) = 1/2.

8.(b) $\lim_{x \to -1^+} f(x) = +\infty$.

To deduce this limit, note that the denominator of f can be factored:

$$f(x) = \frac{1-x}{1-x^2} = \frac{1-x}{(1-x)\cdot(1+x)} = \frac{1-x}{1-x}\cdot\frac{1}{1+x}.$$

When x is slightly smaller than −1, $f(x)$ will look like:

$$f(x) = \frac{1-x}{1-x}\cdot\frac{1}{1+x} \approx \frac{(\text{Approximately } 2)}{(\text{Approximately } 2)}\cdot\frac{1}{(\text{Very small negative})} = \text{Very big negative}.$$

8.(c) $\lim_{x \to 0} g(x)$ does not exist because the left and right hand limits of $g(x)$ at $x = 0$ are not equal. From the graph of $g(x)$ that was given, you can see that the left hand limit is equal to +1 whereas the right hand limit is equal to +2.

8.(d) $\mathit{Lim}_{x \to 2} \frac{f(x)}{g(x)}$ does not exist because the left and right hand limits of this quotient are not equal at $x = 2$. The left hand limit is equal to $+\infty$ whereas the right hand limit is equal to $-\infty$. We will give the analysis for the right hand limit in complete detail below; the determination of the left hand limit can be completed via a very similar analysis.

To deduce the value of the right hand limit, you can look at the behavior of the functions $f(x)$ and $g(x)$ as each approach $x = 2$ from the right, and then combine these to understand how the quotient $\frac{f(x)}{g(x)}$ behaves as x approaches 2 from the right.

Behavior of $f(x)$ as x approaches 2 from the right.

The function f is defined when $x = 2$. Its value (which is equal to the limit from the right) is: $f(2) = 1/3$. Therefore, when x is a number slightly larger than 2, $f(x)$ will be approximately equal to 1/3.

Behavior of $g(x)$ as x approaches 2 from the right.

Inspection of the graph of g shows that when x is a number slightly larger than 2, $g(x)$ is a negative number that is very, very close to zero.

Behavior of $\frac{f(x)}{g(x)}$ as x approaches 2 from the right.

Combining the previous two observations, you have that when x is a number slightly greater than 2,

$$\frac{f(x)}{g(x)} \approx \frac{\frac{1}{3}}{(\text{Small negative})} = \text{Large negative}.$$

So, as x gets closer and closer to 2 from the right, $\frac{f(x)}{g(x)}$ is a larger and larger negative number.

9.(a) $12x - y = 16$

9.(b) $x + y = 3$

9.(c) $5x - y = 10$

9.(d) $18x - y = -25$

10.(a) $A(w) = \frac{-6}{5}w^2 + 18w$

10.(b) $w = 7.5$ miles and $L = 9$ miles.

10.(c) The area is 67.5 square miles, which will support 1350 cattle. The 3 fields will support the herd.