## Unit Test 1 Review Problems – Set A

We have chosen these problems because we think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the exam will resemble these problems in any way whatsoever. Remember that on exams you will have to supply evidence for your conclusions and may have to explain why your answers are reasonable and appropriate.

1.   Find formulas for each of the following functions. Your formulas should not contain any unspecified constants. You can assume that (a) is the graph of a polynomial function, and (b) is part of the graph of a periodic function.

(a)
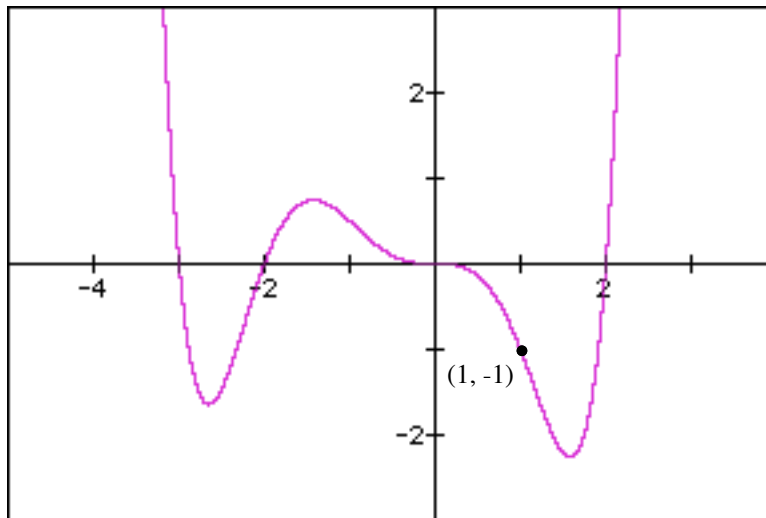
(b)
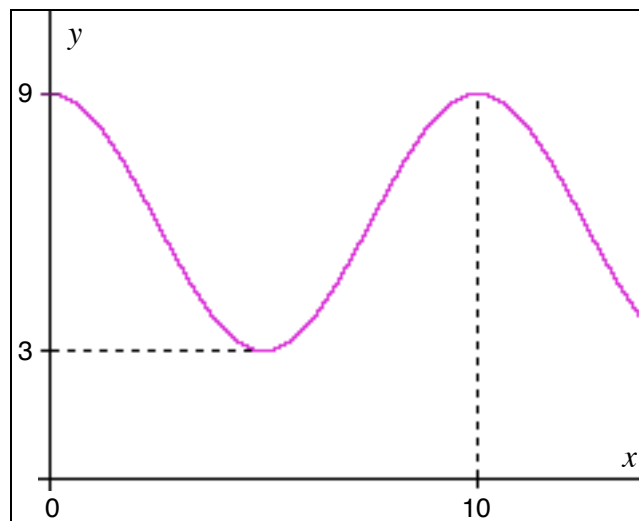
2. The table given below shows some of the values of the functions $f(x)$, $g(x)$ and $k(x) = g(f(x))$. Use the information in the table to evaluate the expressions given in Parts (a)-(f) of this problem.
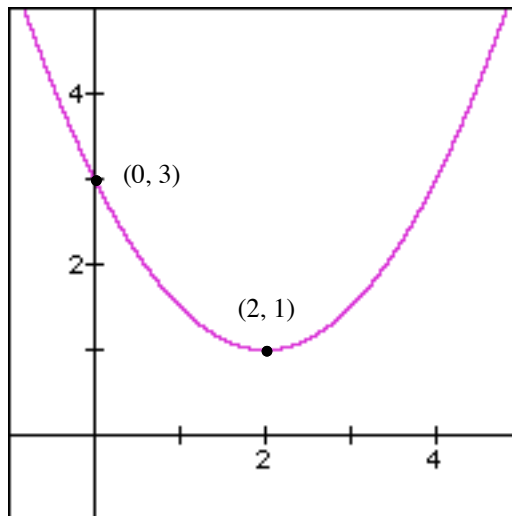
x	$f(x)$	$g(x)$	$k(x) = g(f(x))$
0	4	4	
2	0		4
4	5	2	
5		10	4
6	2		8
8	6	0	6
10		3	0

(a) $k(0)$ **(b)** $k(4)$ **(c)** $g(2)$

**(d)** $g(6)$ **(e)** $f(5)$ **(f)** $f(10)$

(g) A function $p(x)$ is defined by the formula: $p(x) = x^2 + 3$. Use the information contained in the table above to evaluate and simplify the following expression:

$$\frac{p(k(5) + h) - p(k(5))}{h}.$$

3. **(a)** Find a formula for the quadratic function graphed below. Your answer should not contain any unspecified symbols (like a).

$(0, 3)$

$(2, 1)$

(b)   The diagram given below shows a unit circle (that is, a circle with a radius of 1) whose center is located at the point (0, 0). The line T is a tangent line that touches the circle at one point only.   Note that the lines T and L are perpendicular. The diagram is **not** drawn to scale so you should not use it to try to estimate an answer.
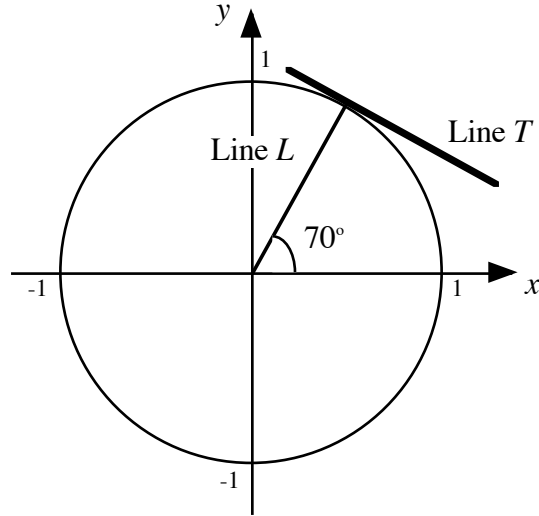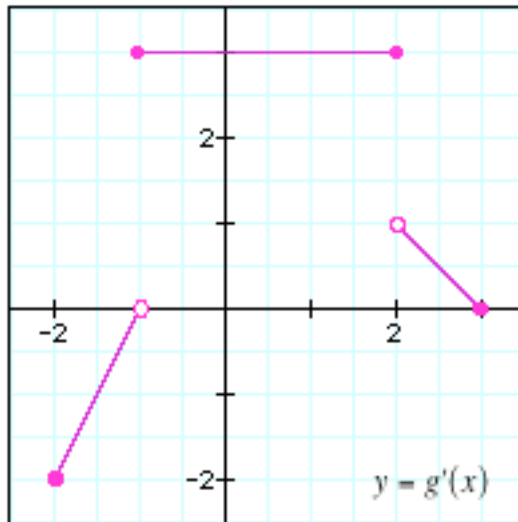

Calculate the *exact* value of the *x*-intercept of the line T.

4.   In this problem, the functions f and g will always refer to the function defined by the equation:

$$f(x) = \tfrac{1}{2}x^3 - x^2 + 3\pi,$$

and the function whose derivative is graphed below.

$y = g'(x)$

(a)   Using any rules that you know for finding derivatives, find an equation for $f'(x)$.

(b)   Let *m* be the function defined by the equation: $m(x) = f(x) + g(x)$. Find the value of $m'(0)$.

(c)   Let *r* be the function defined by the equation: $r(x) = m'(x)$. Find an equation for $r(x)$ that is valid on the interval $0 \le x \le 1$.

5.   Felipe and Katherine are the only two candidates in an election. There are a total of 1000 people who can vote in the election. A person can decide to vote for Felipe, decide to vote for Katherine or be undecided. Let *t* be the number of weeks that Felipe and Katherine have been running their election campaigns. The election is held when $t = 15$.

For the curved part of the graph:

$$F(t) = 16t^2 - 240t + 1154$$
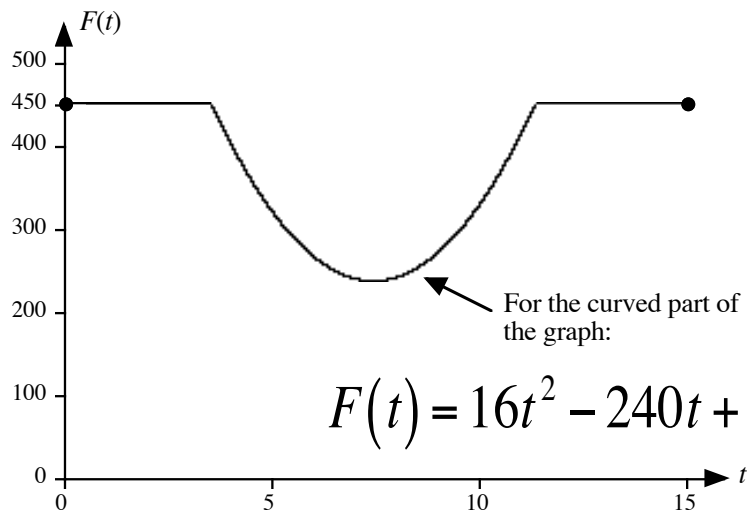
The number of voters who have decided to vote for Felipe by week *t* is given by a function $F(t)$ which is graphed above. Note that $F(0) = F(15) = 450$.

(a)   Write down a formula for $F(t)$, expressing it as a function defined in pieces.

(b)   At time $t = 0$ there are 270 undecided voters. Each week, 15 of the remaining undecided voters decide who they are going to vote for. Find a formula for $U(t)$, the number of undecided voters remaining after *t* weeks.

(c)   Assuming that all of the people who decided to vote for Felipe at $t = 15$ actually do vote for him, and all of the people who decided to vote for Katherine at $t = 15$ actually do vote for her, and the people who are undecided at $t = 15$ do not vote, who wins the election? Using appropriate calculations, briefly explain your answer.

6.   In this problem, the function $f(x)$ will always refer to the function defined by the equation:

$$f(x) = \frac{1-x}{x^2},$$

with domain $x > 0$.

(a) Complete the following table. What is the value of the derivative of $f(x)$ at $x = 3$?

x	2.9	2.99	2.999	2.9999	3.0001	3.001	3.01	3.1
$\dfrac{f(x) - f(3)}{x - 3}$								

(b) Find the equation of the tangent line of $f(x)$ that is based at $x = 3$. Sketch a graph showing both $y = f(x)$ and the tangent line based at $x = 3$. Indicate how the result of Part (a) influences the appearance of your graph.

(c) Use the definition of the derivative to find an equation for the derivative of $f(x)$ at $x = a$.

(d) Sketch a graph of $y = f(x)$ and directly below that, a graph of $y = f'(x)$.

7. Find each of the limits listed below. If you do not believe that a certain limit exists, write "DOES NOT EXIST."

(a) $\displaystyle \lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}.$

(b) $\displaystyle \lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}.$

(c) $\displaystyle \lim_{t \to 0} \frac{\sqrt{t + 4} - 2}{t}.$

(d) $\displaystyle \lim_{x \to 4} \frac{x^2 - 16}{2 - \sqrt{x}}.$

8. In this problem, the function f will always refer to the function defined by the equation:

$$f(x) = \frac{1 - x}{1 - x^2}$$

and the function g will always refer to the function defined by the graph:
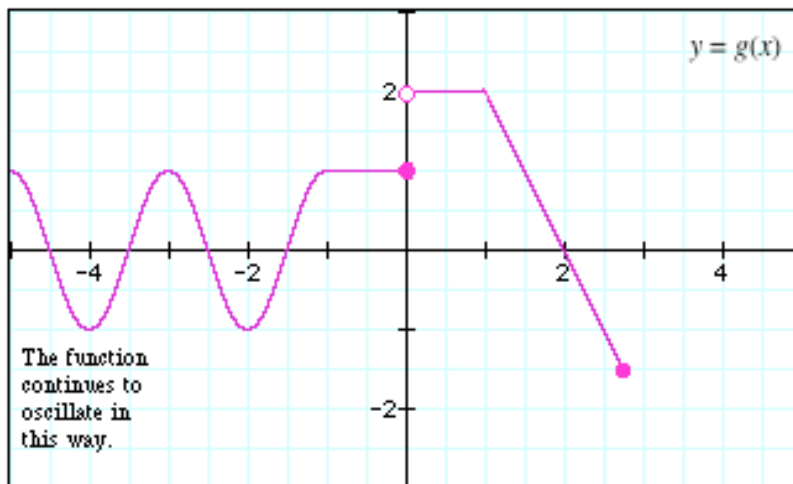
In each of the following cases decide whether or not the limit (possibly a left- or a right-hand limit, check the notation carefully) exists. If you believe that a limit exists, determine its value. If you believe that a limit does not exist, give your reason.

(a) $\underset{x \to 1}{\text{Lim}} \ f(x)$

(b) $\underset{x \to -1^+}{\text{Lim}} \ f(x)$

(c) $\underset{x \to 0}{\text{Lim}} \ g(x)$

(d) $\underset{x \to 2}{\text{Lim}} \ \dfrac{f(x)}{g(x)}$

9. For each of the curves listed below, find the equation of the tangent line at the point (x, y) given.

(a) $y = x^3$ at the point (2, 8).

(b) $y = \dfrac{1}{x-1}$ at the point (2, 1).

(c) $y = x^3 + 3x^2 - 4x - 5$ at the point (1, –5).

(d) $y = \dfrac{3}{x^2} - \dfrac{4}{x^3}$ at the point (–1, 7).

10. A rancher wants to fence in some grazing land in the pattern shown here. (Note that this diagram is not drawn to scale – the heights of the fences surrounding the grazing land are much higher in this diagram than they would be in reality, for example.)

The rancher plans to create one large field that will take up half the total area. The other half of the land will be divided by a fence into two smaller fields. She has enough funds to buy 45 miles of fencing material.
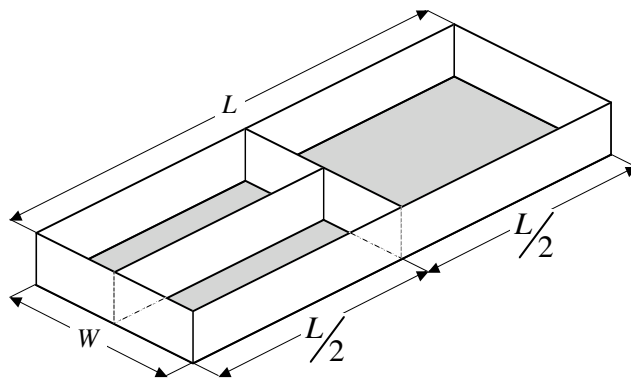
(a) Let $A(W)$ represent the total area of the three fields as a function of width, W, only. Find a formula for $A(W)$.

(b)   Find the values of L and W that the rancher should use to maximize the area of land covered by the three fields.   Do not use your calculator except for arithmetic. Show your all of your work and all of the steps in your calculation. Give appropriate units with your answer.

(c)   The rancher's land can sustain 20 cattle per square mile. If she owns a herd of 1300 cattle, will it be possible for the three fields to sustain them all? Explain why or why not with the aid of a calculation.