Outline

1. Infinite limits.
2. Tangent lines and derivatives.
3. Calculating derivatives.

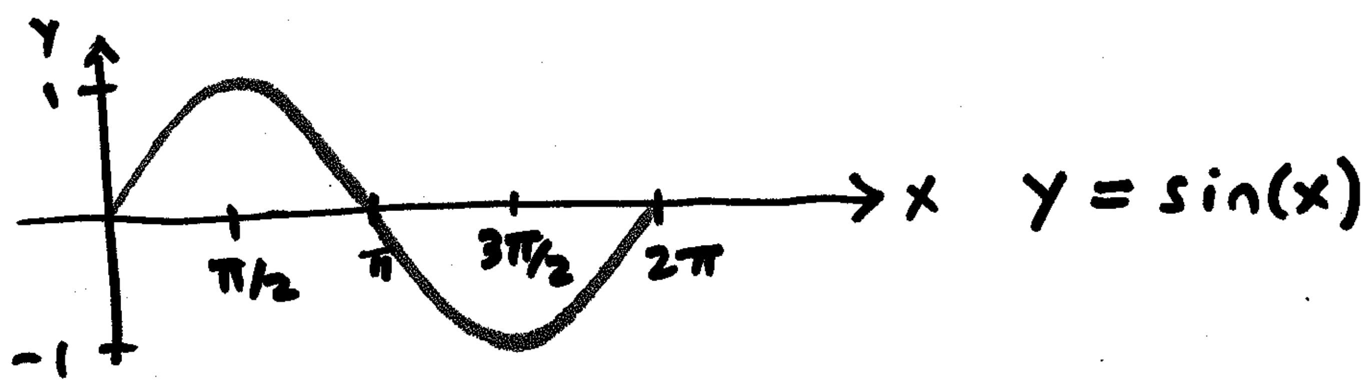
1. Infinite Limits

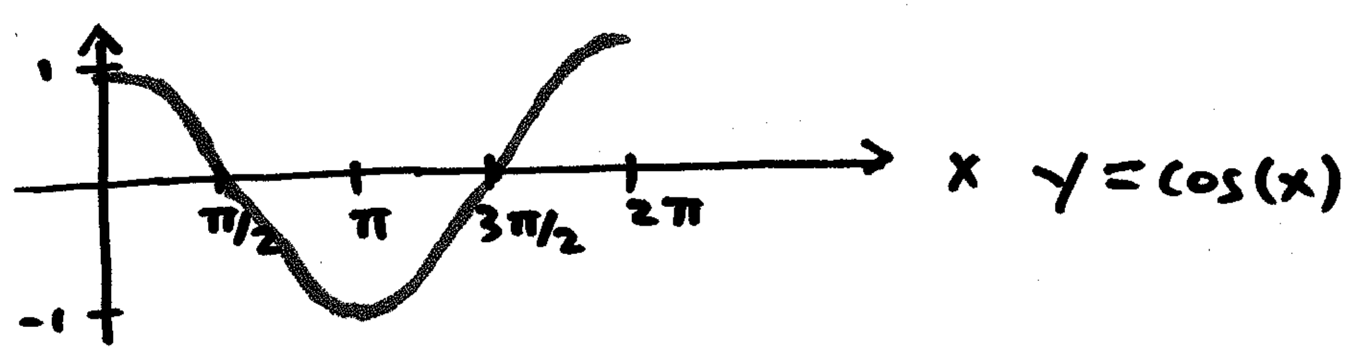
Example

What is $\lim_{x \to \frac{\pi}{2}} \tan(x)$ equal to?

Solution

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$





When x is close to $\pi/2$ ...

... $\sin(x)$ is close to 1.

... $\cos(x)$ is close to 0.

$x < \pi/2$ means $\cos(x) > 0$. $(+)$

$x > \pi/2$ means $\cos(x) < 0$. $(-)$

- when x is close to $\pi/2$ but slightly to the left of $\pi/2$,

$$\tan(x) = \text{[redacted]} \frac{\sin(x)}{\cos(x} = \frac{\text{Approx } 1}{\text{Small positive number}}$$

$$= \text{very large positive number.}$$

- In limit notation:

$$\lim_{x \to \frac{\pi}{2}^-} \tan(x) = +\infty.$$

- when x is close to $\pi/2$ but slightly to the right of $\pi/2$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\text{Approx. } 1}{\text{small negative number}}$$

$$= \text{very large negative number}$$

- In limit notation:

$$\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty.$$

- Final answer:

$$\lim_{x \to \frac{\pi}{2}} \tan(x) \text{ does not exist.}$$

Warning:

The ∞ symbol is not a number and can't be manipulated like one.

e.g. What is $\lim_{x \to \infty} x^3 - 2x$?

Possible answers: (a) $\infty^3 - 2\infty$

(b) ∞

(c) $-\infty$

(d) Does not exist.

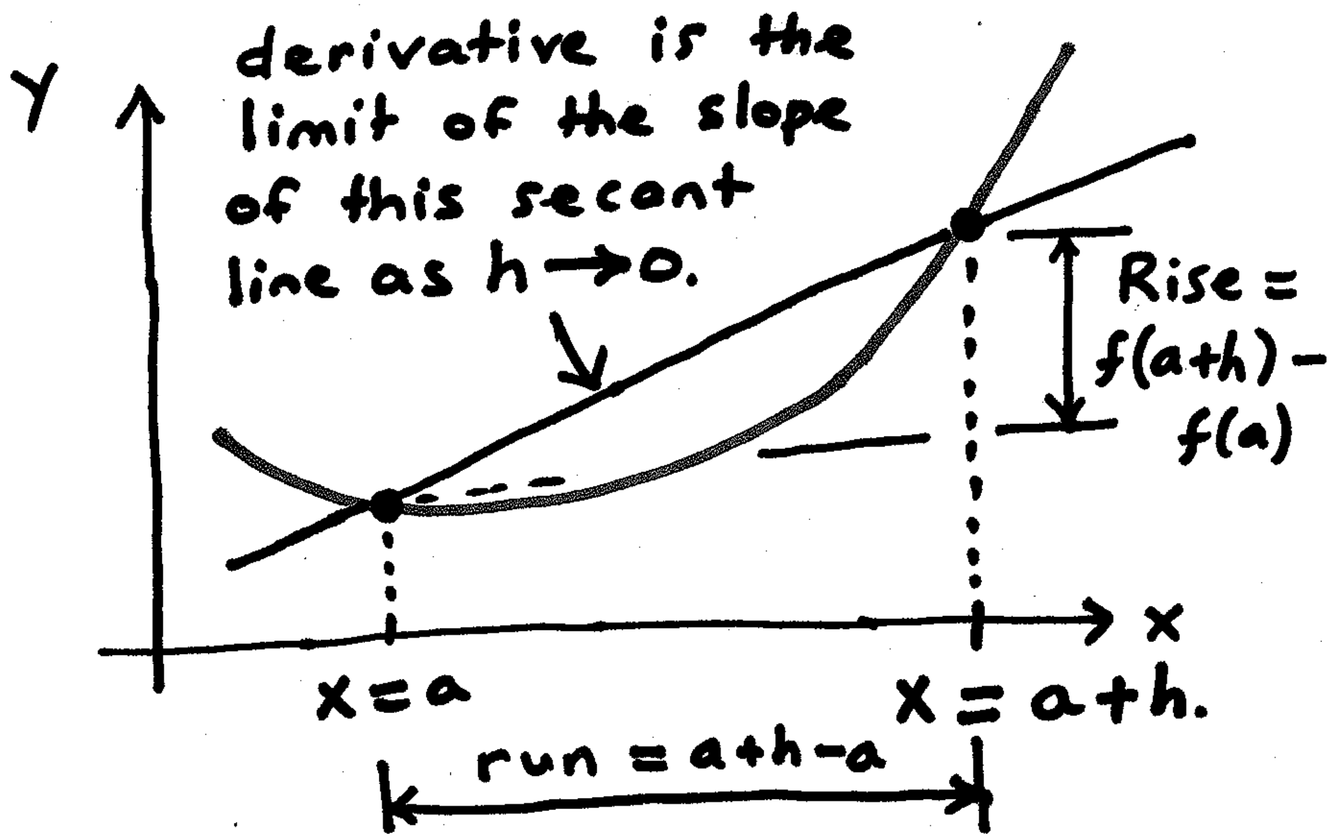
Correct answer is (b). (or (d) according to book, but (b) is preferred).

- Can't get the right answer by plugging ∞ into the formula – SO DON'T!!!

2. Derivatives and Tangent Lines

- The derivative of $f(x)$ at a point $x = a$ (normally written as $f'(a)$) is the limit:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



- The geometrical meaning of $f'(a)$ is that it is the slope of the tangent line to the graph $y = f(a)$ at the point $x = a$.

- It is also said ... $f'(a)$ is the instantaneous rate of change of the function $f(x)$ at the point $x = a$.

3. <u>Calculating</u> <u>Derivatives</u>

<u>Example</u>

Calculate $f'(4)$ for $f(x) = x^3$.

Solution

$$\frac{f(4+h)-f(4)}{h} = \frac{(4+h)^3 - 4^3}{h}$$

$$= \frac{4^3 + 3\cdot 4^2\cdot h + 3\cdot 4\cdot h^2 + h^3 - 4^3}{h}$$

$$= \frac{h\cdot(3\cdot 4^2 + 3\cdot 4\cdot h + h^2)}{h}$$

$$= 3\cdot 4^2 + 3\cdot 4\cdot h + h^2, \quad h \neq 0$$

$$f'(4) = \lim_{h\to 0} 3\cdot 4^2 + 3\cdot 4\cdot h + h^2$$

$$= 3\times 4^2 + (3)(4)(0) + 0^2$$

$$= 3\times 4^2 = 48.$$