Outline

1. When limits do & do not exist.
2. Calculating limits.
3. Squeeze Lemma
4. Limits involving infinity.

—||—

HW #2 due at start of recitation.

1. When Limits Do and Do Not Exist.

Example

Does $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ exist or not?

Solution

- limit from left: $\lim\limits_{x \to 0^-} \dfrac{|x|}{x}$

x	-0.1	-0.01	-0.001	-0.0001
$\dfrac{\lvert x\rvert}{x}$	-1	-1	-1	-1

Suspect: $\lim\limits_{x \to 0^-} \dfrac{|x|}{x} = -1.$

- Limit from right:

x	0.1	0.01	0.001	0.0001
$\frac{\lvert x\rvert}{x}$	1	1	1	1

Suspect: $\lim_{x\to 0^+} \frac{|x|}{x} = 1.$

So, since $\lim_{x\to 0^-} \frac{|x|}{x}$ does not equal $\lim_{x\to 0^+} \frac{|x|}{x}$, the overall limit

$$\lim_{x\to 0} \frac{|x|}{x} \text{ does not exist.}$$

To be 100% sure...

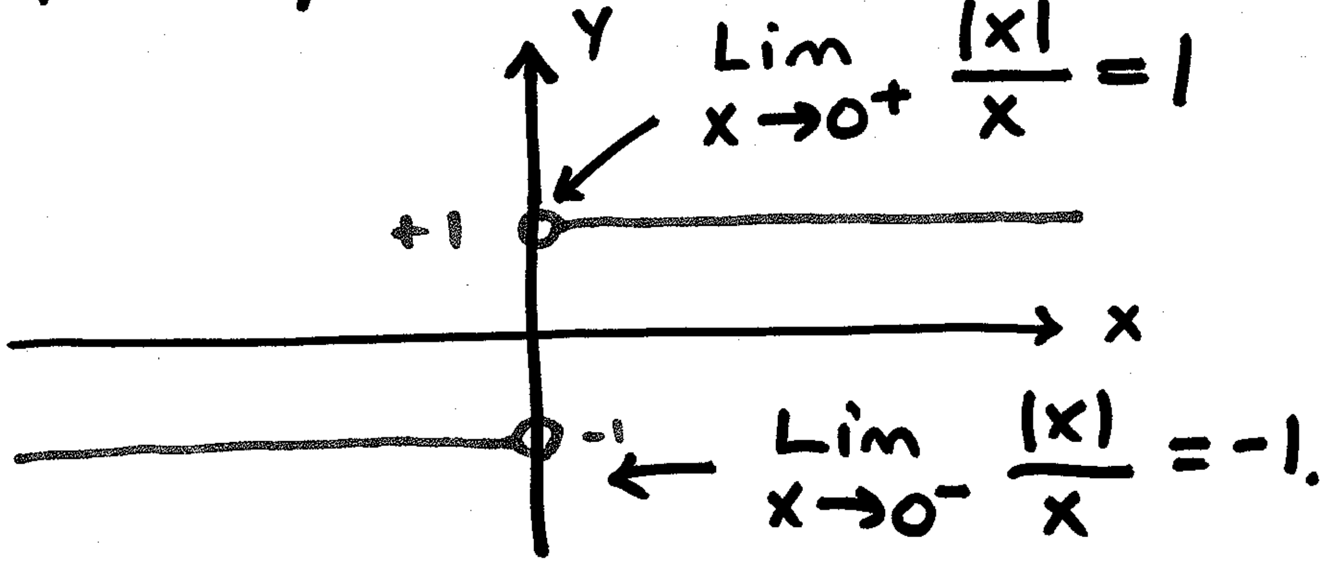
when $x > 0$, $|x| = x$ so

$$\frac{|x|}{x} = 1. \qquad x > 0.$$

when $x < 0$, $|x| = -x$ so

$$\frac{|x|}{x} = \frac{-x}{x} = -1 \qquad x < 0$$

graphically:



2. Calculating Limits

- If you have to calculate $\lim\limits_{x \to a} f(x)$ and 'a' is in the domain of $f(x)$, then:

$$\lim_{x \to a} f(x) = f(a)$$

(provided the example isn't set up to deliberately mess this up).

Example

Calculate $\lim_{x \to 2} f(x)$ where:

(a) $f(x) = x^2$

(b) $f(x) = \begin{cases} x^2, & x < 2 \\ 19, & x = 2 \\ x^2, & x > 2 \end{cases}$

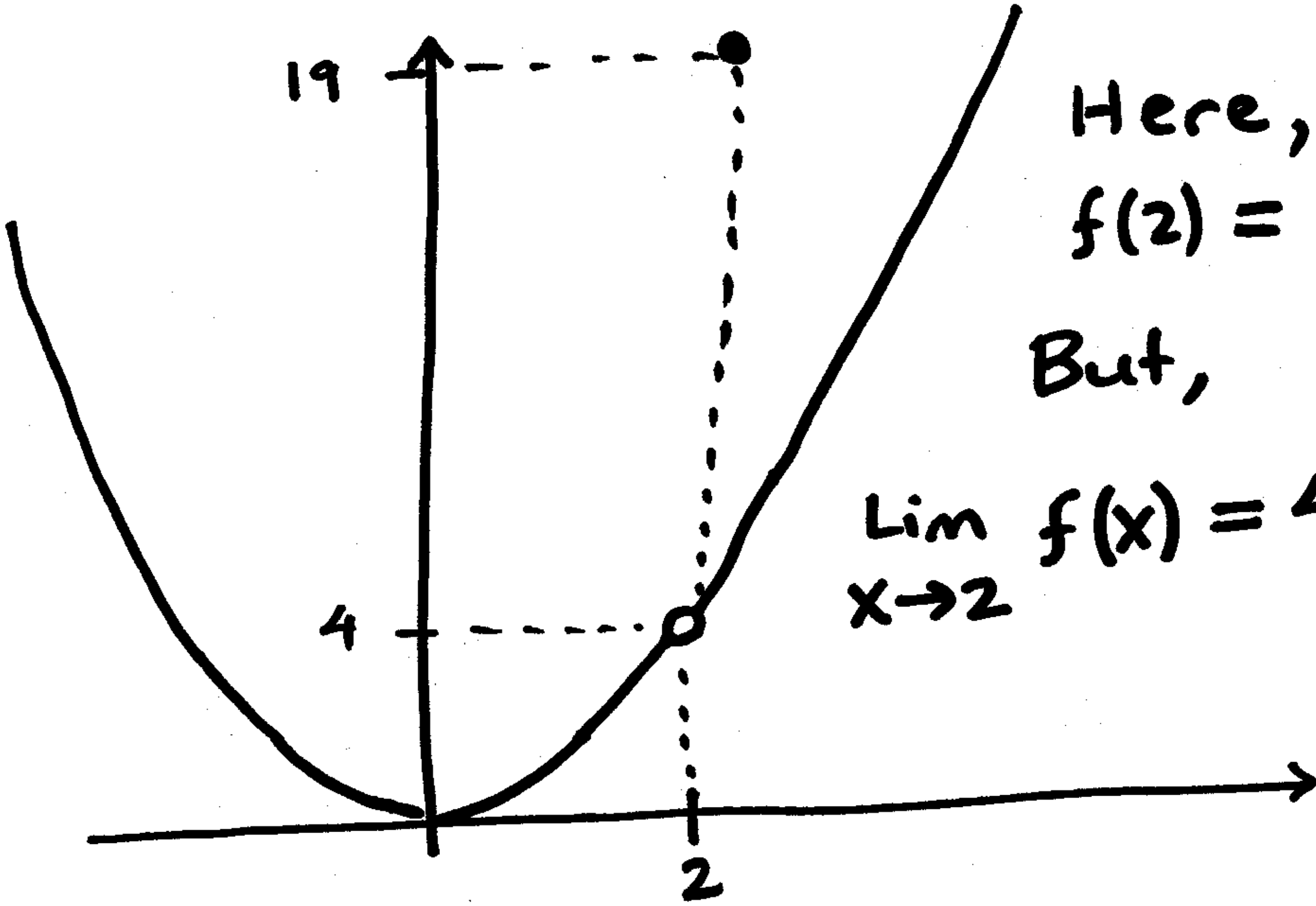
Solution

(a) $\lim_{x \to 2} f(x) = 2^2 = 4.$

$\uparrow$

$f(2) = 2^2 = 4$

(b) If we graph $y = f(x)$, we get:



Here, $f(2) = 19$.

But, $\lim_{x \to 2} f(x) = 4$.

Note: A function $f(x)$ which has the property that for $x = a$:

$$\lim_{x \to a} f(x) = f(a)$$

$$\text{Limit} = \text{function value}$$

is said to be <u>continuous</u> at $x = a$.

- <u>Rule of thumb:</u> The graph $y = f(x)$

is the graph of a continuous function if you can draw the graph without lifting your pencil from the page.

- If you have to calculate $\lim_{x \to a} f(x)$ and $x = a$ is not in the domain of $f(x)$, then:

  ① Simplify the $f(x)$ formula as much as possible assuming $x \neq a$ to get a simpler formula $g(x)$.

  ② Calculate $\lim_{x \to a} g(x)$. This is equal to $\lim_{x \to a} f(x)$.

Example

Calculate $\displaystyle\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$.

Solution

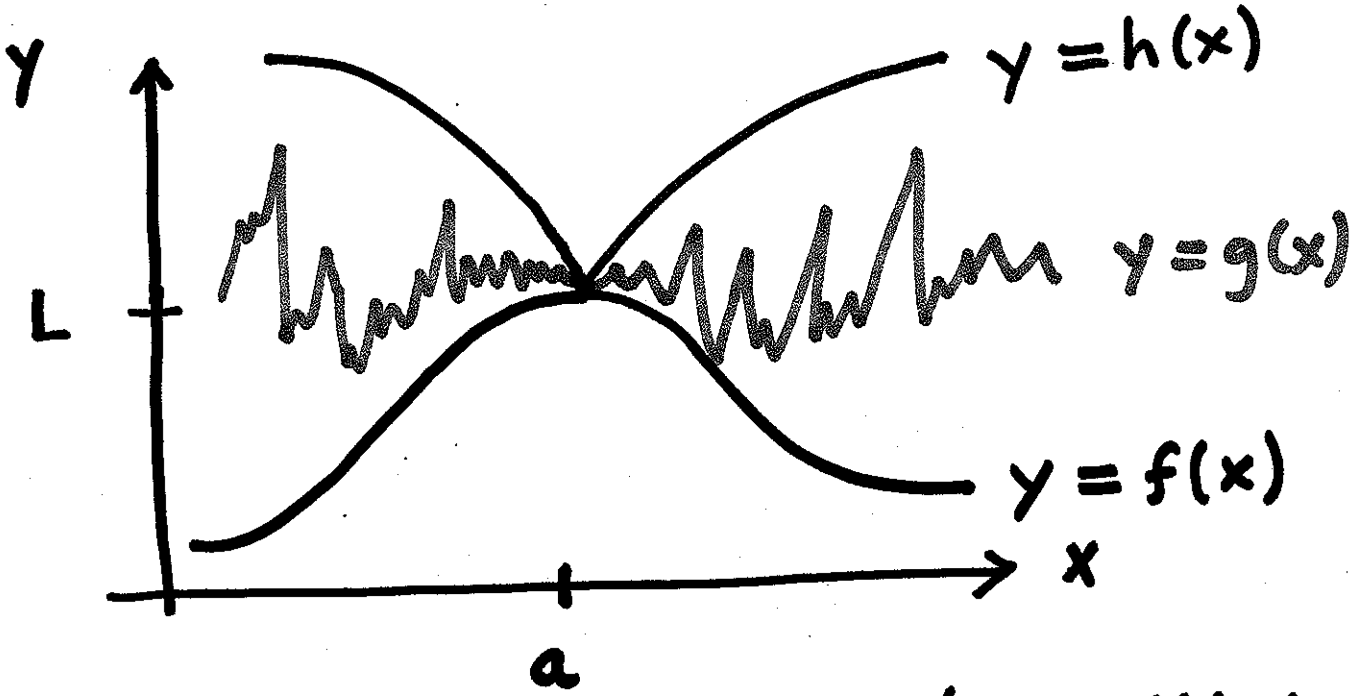
$$f(x) = \frac{x^2 - 4x}{x^2 - 3x - 4} \qquad x = 4.$$

$$= \frac{x(x-4)}{(x-4)(x+1)} \qquad \text{If } x \neq 4$$

$$g(x) = \frac{x}{x+1}$$

$$\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{5}.$$

3. Squeezing Lemma



Want: $\lim_{x \to a} g(x)$. (but it's too hard to calculate).

If $f(x) \leq g(x) \leq h(x)$ and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then $\lim_{x \to a} g(x) = L$ also.