# Outline

1. Sine and cosine
2. What are limits and why do we need them?
3. When limits do and do not exist.

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Quiz tomorrow. Review problems on-line.

## 1. Functions Based on Sine and Cosine

- Period = horizontal distance from peak to peak.

- Amplitude = distance (vertical) from midline to peak

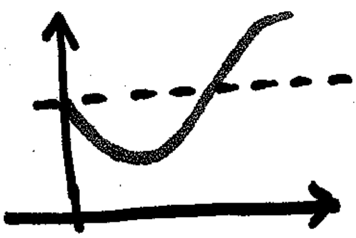
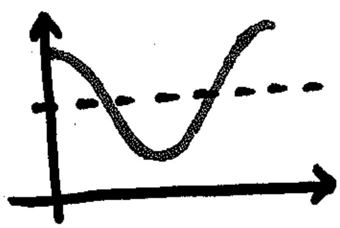
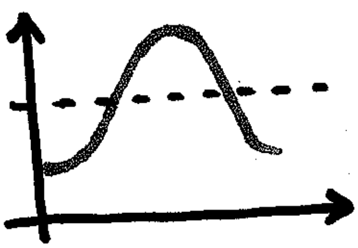
$$= \frac{\text{max y-value} - \text{min. y-value}}{2}$$

- Midline = y-value that splits the graph in two

$$= \frac{\text{max. y-value} + \text{min. y-value}}{2}$$

A = amplitude
M = midline
P = period.

- There are 4 basic functions based on sine (sin) or cosine (cos).

| Behavior near y-axis (picture) | Behavior near y-axis (words) | Formula. |
|---|---|---|
|  | Graph starts at midline then rises. | $y = A \cdot \sin\left(\frac{2\pi}{P}x\right) + M$ |
|  | Graph starts at midline then decreases. | $y = -A \cdot \sin\left(\frac{2\pi}{P}x\right) + M$ |
|  | Graph starts at peak then decreases. | $y = A \cdot \cos\left(\frac{2\pi}{P}x\right) + M$ |
|  | Graph starts at bottom of trough then rises. | $y = -A \cdot \cos\left(\frac{2\pi}{P}x\right) + M$ |

Example

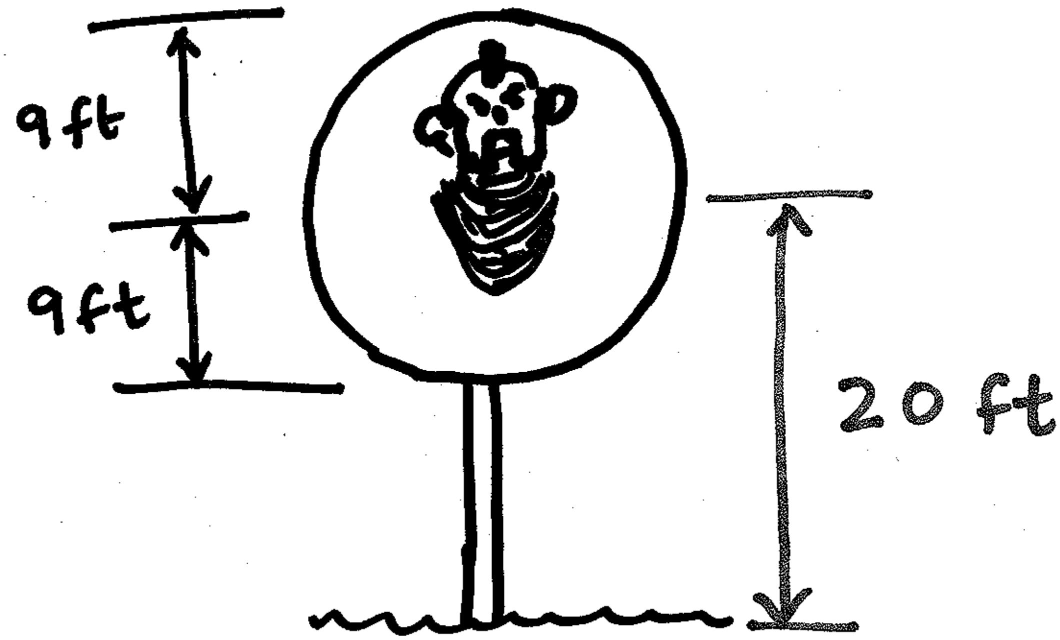
Mr T. is strapped to a spinning circle/wheel. The diameter of the wheel is 18 ft and Mr. T's head is right on the edge of the wheel. The center of the wheel is 20 ft off the ground. Mr. T starts in the 12 o'clock position and the wheel completes one rotation every 15 seconds.

~~[illegible]~~ $h(t)$ is height of Mr. T's head above the ground $t$ minutes after wheel starts spinning.

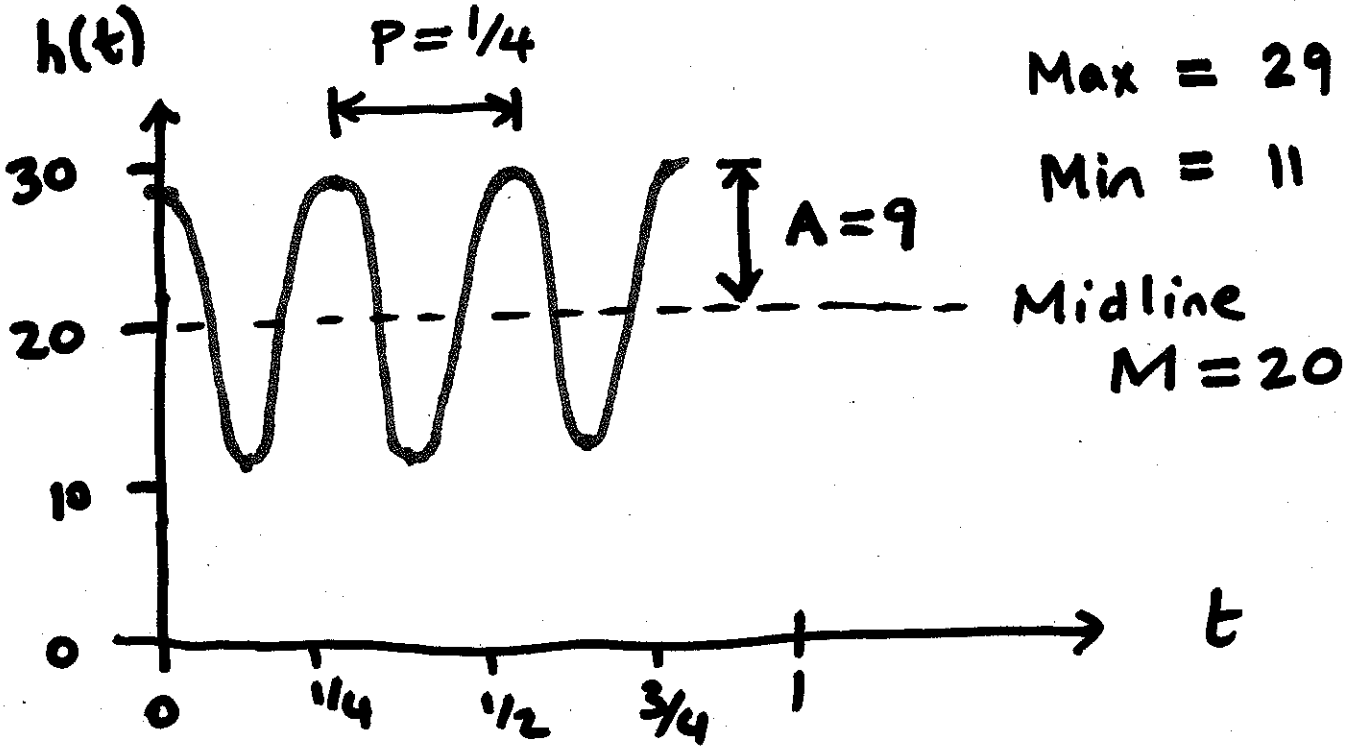
(a) Graph $h(t)$.

(b) Find a formula for $h(t)$.

# Solution




(a)




(b) $h(t) = 9 \cdot \cos\left(\frac{2\pi}{1/4} t\right) + 20.$

## 2. Why Do We Have Limits?

- Limits help us probe the behavior of functions at x-values where normal arithmetic and graphs break down.

**Example**

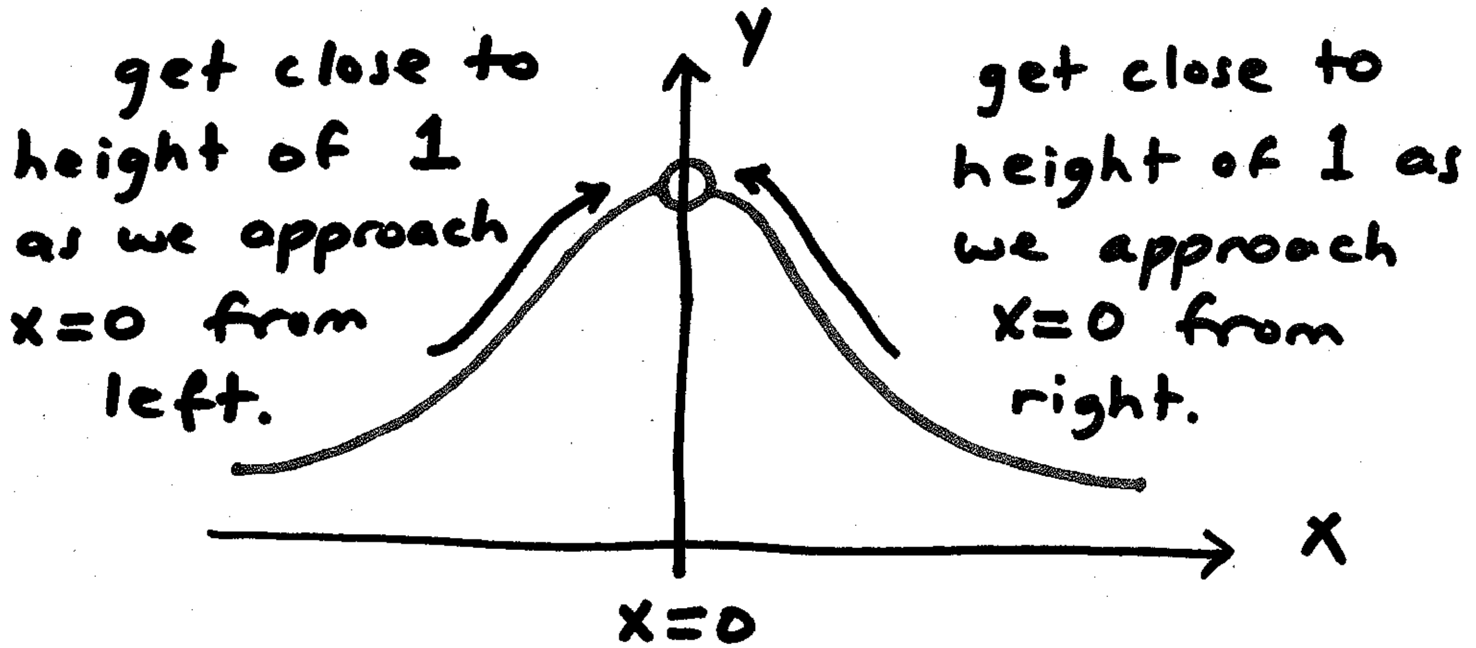
Consider $f(x) = \dfrac{\sin(x)}{x}$.

$x = 0$ is not in the domain so we can't work out $f(0)$.

The limit of $f(x)$ as $x$ approaches zero, written:

$$\lim_{x \to 0} f(x)$$

is the value we would assign to $f(0)$ if it were possible to calculate $f(0)$.




The limit of f(x) as x → 0 is:

$$\lim_{x \to 0} f(x) = 1.$$