Outline

1. u-substitution.

2. Integration by parts.

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Reminder: ① Quiz Tuesday.

② Do-over: Wednesday
1212 DH
8-9pm or 9-10pm

③ Final: Friday May 8
1-4 pm A51 BH

1. u-substitution

$$\int \left(\ln(x)+1\right)\cdot \frac{1}{\sqrt{x\cdot \ln(x)}}\,dx.$$

① Try $u = x\cdot\ln(x)$

② $\dfrac{du}{dx} = \ln(x) + x\cdot\dfrac{1}{x} = \ln(x)+1.$

③ $dx = \dfrac{du}{\ln(x)+1}$

④ $\displaystyle\int (\ln(x)+1)\cdot\frac{1}{\sqrt{u}}\cdot\frac{du}{\ln(x)+1} = \int u^{-1/2}\,du$

$$= \frac{u^{1/2}}{1/2} + C$$

⑤ $\displaystyle\int (\ln(x)+1)\cdot\frac{1}{\sqrt{x\cdot\ln(x)}}\,dx = 2\sqrt{x\cdot\ln(x)}+C$

Example

$f(v) = \frac{1}{v}$

$$\int \frac{1}{x} \cdot \frac{1}{\ln(x)}\, dx$$

① $u = \ln(x)$

② $\frac{du}{dx} = \frac{1}{x}$

③ $dx = x \cdot du$

④ $$\int \frac{1}{x} \cdot \frac{1}{\ln(u)}\, dx = \int \frac{1}{x} \cdot \frac{1}{u} \cdot x\, du$$

$$= \int \frac{1}{u}\, du$$

⑤ $$= \ln(|u|) + C$$

$$= \ln(|\ln(x)|) + C$$

Example

$$\int \left(\frac{-1}{x^2}+2x\right)\sqrt{\frac{1}{x}+x^2+1}\ dx$$

① $u = \frac{1}{x}+x^2+1$

② $\frac{du}{dx} = \frac{-1}{x^2}+2x$

③ $dx = \frac{du}{\frac{-1}{x^2}+2x}$

④ $\int \left(\frac{-1}{x^2}+2x\right)\sqrt{\frac{1}{x}+x^2+1}\,dx =$

$$\int \left(\frac{-1}{x^2}+2x\right) u^{1/2}\cdot\frac{du}{\frac{-1}{x^2}+2x} = \int u^{1/2}du$$

$$= \frac{2}{3}u^{3/2}+C$$

⑤ $$= \frac{2}{3}\left(\frac{1}{x}+x^2+1\right)^{3/2}+C$$

2. Integration by Parts

- Very useful for integrating products of two functions when u-substitution doesn't work.

- Formula:

$$\int u \cdot v' \, dx = uv - \int u' \cdot v \, dx$$

Example

Evaluate: $\int x \cdot \cos(x) \cdot dx.$

$u = x$ $\qquad$ $v' = \cos(x)$

$u' = 1$ $\qquad$ $v = \sin(x)$

Solution

$$\int u \cdot v' \, dx = u \cdot v - \int u' \cdot v \, dx$$

$$\int x \cdot \cos(x) \, dx = x \cdot \sin(x) - \int 1 \cdot \sin(x) \, dx$$

$$= x \cdot \sin(x) - -\cos(x) + C$$

$$= x \cdot \sin(x) + \cos(x) + C$$

Check:

$$\frac{d}{dx}\left(x \cdot \sin(x) + \cos(x) + C \right)$$

$$= \sin(x) + x \cdot \cos(x) - \sin(x) + 0$$

$$= x \cdot \cos(x) \quad \checkmark$$