Outline

1. Rules for manipulating integrals.
2. U-substitution.

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Final Exam: Friday May 8
1pm - 4pm

Third Unit Test: Friday April 24
10:30 am.

1. Rules for Manipulating Integrals

- **Additive Laws:**

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b \big(f(x) + g(x) \big)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

- **Constant Law:**

$$\int_a^b \underset{\substack{\uparrow \\ \text{constant}}}{c} \cdot f(x)\,dx = c \cdot \int_a^b f(x)\,dx$$

- **Subtractive / Reversal Laws:**

$$\int_a^b \big(f(x) - g(x) \big)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

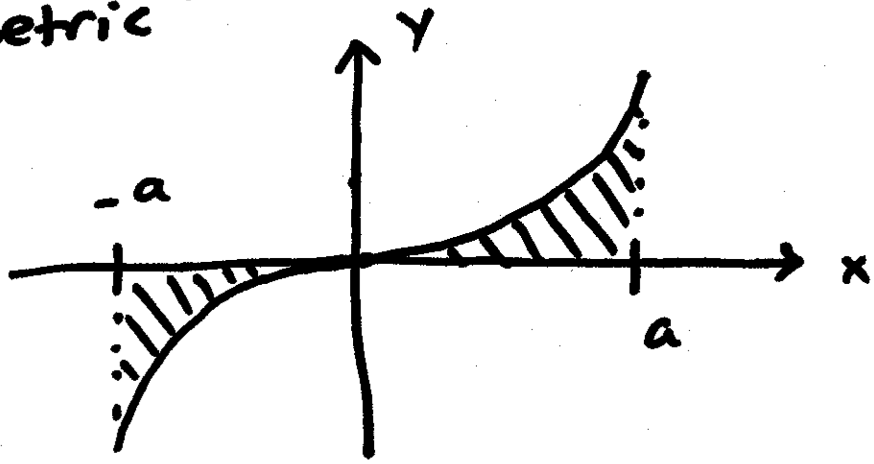
order of limits reversed.

- **Odd and Even Laws:**

$f(x)$ is odd: $f(-x) = -f(x)$.

$f(x)$ is odd: $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

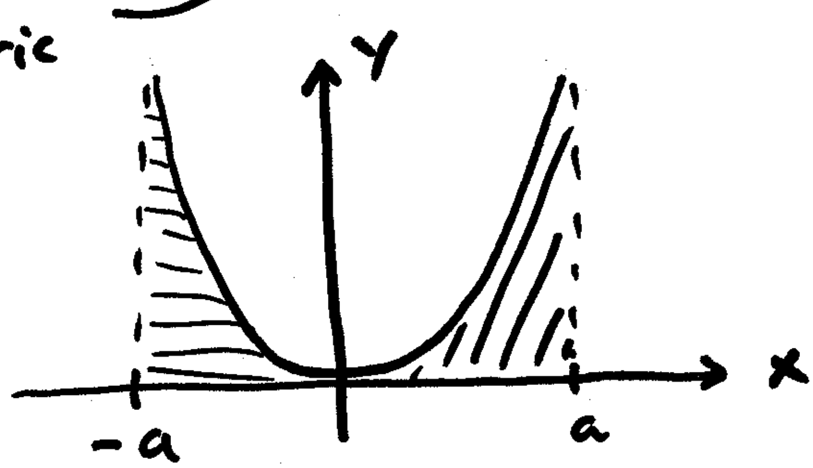
interval must be symmetric



$f(x)$ is even: $f(-x) = f(x)$.

$f(x)$ is even: $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\cdot\int_{0}^{a} f(x)\,dx$

interval must be symmetric



2. u-Substitution

- Way to find anti-derivatives by reversing the Chain Rule.

Example

Evaluate the indefinite integral:

$$\int \frac{\frac{2}{3}x + 1}{x^2 + 3x + 7}\,dx.$$

Solution

① Look at the formula you are integrating and find one function inside another. Call the inside function "u."

$$\frac{\frac{2}{3}x + 1}{(x^2 + 3x + 7)}$$

outside function

denominator is the inside function.

$$u = x^2 + 3x + 7.$$

② Calculate $\frac{du}{dx}$.

$$\frac{du}{dx} = 2x + 3$$

③ Solve the du/dx equation to make dx the subject.

$$dx = \frac{du}{2x+3}.$$

④ Plug this into original integral in the place of dx.

$$\int \frac{\frac{2}{3}x + 1}{x^2 + 3x + 7}\, dx = \int \frac{\frac{2}{3}x + 1}{x^2 + 3x + 7} \frac{du}{2x+3}.$$

⑤ Replace inside function in integral by "u."

$$\int \frac{\frac{2}{3}x+1}{x^2+3x+7}\,dx = \int \frac{\frac{2}{3}x+1}{u} \cdot \frac{du}{2x+3}$$

⑥ Simplify the integral (by legitimate math operations) to eliminate/cancel all remaining x's.

$$\int \frac{\frac{2}{3}x+1}{x^2+3x+7}\,dx = \int \frac{\frac{1}{3}(2x+3)}{u} \frac{du}{2x+3}$$

$$= \frac{1}{3}\int \frac{1}{u}\,du$$

Only constants and u are left. If x's still left (and can't be removed), try another choice for u.

(7) Find the antiderivative using "u" as the variable.

$$\int \frac{\frac{2}{3}x+1}{x^2+3x+7}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$$

$$= \frac{1}{3}\ln(|u|) + C$$

(8) Replace "u" in the anti-derivative.

$$\int \frac{\frac{2}{3}x+1}{x^2+3x+7}\,dx = \frac{1}{3}\ln\left(|x^2+3x+7|\right) + C$$

Example

Evaluate the indefinite integral:

$$\int (4x^3+1)\cdot e^{x^4+x+9}\,dx.$$

Solution

inside.

$$(4x^3+1) \cdot e^{(x^4+x+9)}$$

outside.

$$u = x^4 + x + 9.$$

$$\frac{du}{dx} = 4x^3+1 \qquad dx = \frac{du}{4x^3+1}$$

$$\int (4x^3+1) \cdot e^{x^4+x+9} dx = \int (4x^3+1) e^u \cdot \frac{du}{4x^3+1}$$

$$= \int e^u \cdot du$$

$$= e^u + C$$

$$= e^{x^4+x+9} + C$$